

BC Calculus – Taylor Series Review – Solutions

1. a. $(-\infty, \infty)$ b. $[-7, -1)$ c. $(-5, 2.5)$

2. a. $f(x) = \frac{1}{1+x}; x = \frac{1}{4}$

b. $f(x) = \sin x; x = \pi$

c. $f(x) = e^x; x = \ln 2$

3. a. $1 + 6x + 36x^2 + 216x^3 + \dots; \sum_{n=0}^{\infty} (6x)^n$

b. $(\pi x) - \frac{(\pi x)^3}{3!} + \frac{(\pi x)^5}{5!} + \dots; \sum_{n=0}^{\infty} \frac{(-1)^n (\pi x)^{2n+1}}{(2n+1)!}$

c. $-x^2 + \frac{x^4}{3!} - \frac{x^6}{5!} + \frac{x^8}{7!} + \dots; \sum_{n=0}^{\infty} \frac{(-1)^{n-1} x^{2n+2}}{(2n+1)!}$

d. $4x + 4x^2 + 4x^3 + \dots; \sum_{n=0}^{\infty} 4x^{n+1}$

e. $x - x^2 + \frac{x^3}{2!} - \frac{x^4}{3!} + \dots; \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n!}$

4. a. $1 + (x-2) + (x-2)^2 + (x-2)^3 + \dots; \sum_{n=0}^{\infty} (x-2)^n$

b. $\frac{1}{3} - \frac{1}{9}(x-3) + \frac{1}{27}(x-3)^2 - \frac{1}{81}(x-3)^3 + \dots; \sum_{n=0}^{\infty} \frac{(-1)^n (x-3)^n}{3^{n+1}}$

c. $-(x-\pi) + \frac{(x-\pi)^3}{3!} - \frac{(x-\pi)^5}{5!} + \frac{(x-\pi)^7}{7!} + \dots; \sum_{n=0}^{\infty} \frac{(-1)^{n-1} (x-\pi)^{2n+1}}{(2n+1)!}$

5. a. $\frac{1}{2} - \frac{1}{4}(x-1) + \frac{1}{8}(x-1)^2 + \dots + \frac{(-1)^n (x-1)^n}{2^{n+1}}$

b. $(-1, 3)$

c. $\frac{1}{2} - \frac{1}{4}(x-1) + \frac{1}{8}(x-1)^2 - \frac{1}{16}(x-1)^3$

$f(0.5) \approx \frac{85}{128}$

6. a. $-1 + 2x$

b. $-1 + 2x - \frac{3x^2}{2}$

c. $-1 + 2x - \frac{3x^2}{2} + \frac{2x^3}{3}$

d. $f(.7) \approx -0.106$