Calculator
Tables #3

Name________________
Date__________Period__

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

The temperature fo water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t=0$, the temperature of the water is $55^{\circ}F$. The water is heated for 30 minutes, beginning at time $t=0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

a.) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

$W'(12) = \frac{W(15)-W(9)}{15-9} = \frac{67.9-61.8}{6} \frac{^{\circ}F}{min}$ $1.0166 \frac{^{\circ}F}{min}$ +1: estimate

Temp of water is increasing by rate $1.0166\,^{\circ}F/min$ on the 12th minute.

b.) Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

+1: Interpretation w/ units

+1: value

$\int_0^{20} w'(t)\,dt = w(t)\Big|_0^{20} = w(20)-w(0) = 71-55 = 16^{\circ}F$

The temperature change $16^{\circ}F$ over the 20 minute interval +1: Interpretation with units

c.) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation over estimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

A1

$\frac{1}{20}\left[\underbrace{4(55)+5(57.1)+6(61.8)+5(67.9)}_{\text{+1: Left Riemann Sum}}\right] = 60.79$ +1: approximation

Underestimate because $W(t)$ is an increasing function +1: underestimate with Reason

d.) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t=25$?

$W(25) = W(20) + \int_{20}^{25} W'(t)\,dt$ +1: Integral

$71 + 2.043157$

73.043157 +1: answer

Non-Calculator

Tables #5

Name____________________

Date______________ Pd.___

Train A runs back and forth on an east-west section of railroad track. Train A's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where t is measured in minutes. Selected values for $v_A(t)$ are given in the table.					
t (minutes)	0	2	5	8	12
$v_A(t)$ (meters/minute)	0	100	40	-120	-150

a.) Find the $\underbrace{\text{average acceleration}}_{\text{Avg ROC}}$ of train A over the interval $2 \le t \le 8$.

$$\frac{V(8)-V(2)}{8-2} = \frac{-120-100}{6} = \frac{-220}{6} = \frac{-110}{3}\ \frac{\text{meters}}{\text{min}^2}$$

+1: avg acceleration

b.) Do the data in the table support the conclusion that train A's velocity is -100 meters per minute at some time t with $5 < t < 8$? Give a reason for your answer.

$V_A(8)=-120$ $V_A(5)=40$ +1: $V_A(8) < -100 < V_A(5)$

V_A is continuous & differentiable ∴ by IVT

$V_A(t)=-100$ on $(5,8)$ +1: conclusion using IVT

c.) At time $t=2$, train A's position is 300 meters east of the Origin Station, and the train is moving to the east. Write an expression involving an integral that gives the position of train A, in meters from the Origin Station, at time $t=12$. Use a trapezoidal sum with three subintervals indicated by the table to approximate the position of the train at time $t=12$.

$P(12) = P(2) + \int_2^{12} V_A(t)\,dt$ +1: position expression

$= 300 + \frac{3}{2}[100+40] + \frac{3}{2}[40+{}^-120] + \frac{4}{2}[{}^-120+{}^-150]$ +1: trapezoidal sum

$= 300 + 3(70) - 3(40) - 2(270) = 510 - 120 - 540 = -150$ +1: position

d.) A second train, train B, travels north from the Origin Station. At time t the velocity of train B is given by $v_B(t) = -5t^2 + 60t + 25$, and at time $t=2$ the train is 400 meters north of the station. Find the rate, in meters per minute, at which the distance between train A and train B is changing at time $t=2$.

Know V_A = table $V_A(2)=100$

$V_B(2) = -5(4)+120+25 = 125$

Find: $\frac{dd}{dt} =$

When: $t=2$

$P_A(2)=300$

$P_B(2)=400$

$d(2)=500$

Eqn: $d^2 = P_A^2 + P_B^2$

D: $\cancel{2}d \cdot \frac{dd}{dt} = \cancel{2} \cdot P_A \cdot V_A + \cancel{2} \cdot P_B \cdot V_B$ +1: Implicit derivative of distance

$500\frac{dd}{dt} = 300(100) + 400(125)$

$\frac{dd}{dt} = \frac{300\cancel{00} + 500\cancel{00}}{5\cancel{00}} = \frac{800}{5} = 160$ +1: answer

125
125
250
250
500

160
5 | 800
5
30
30