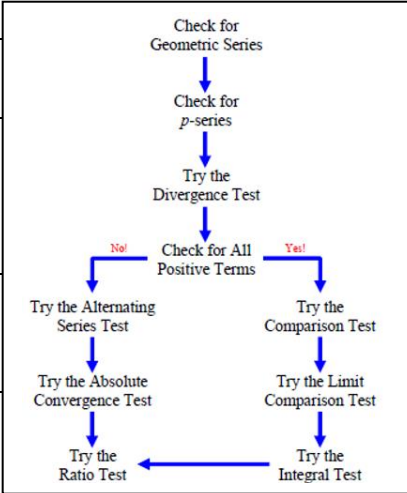


Please start off every review with reading your notecards for that unit several times!!!! This is a very limited review!!!!

Test	What does it look Like?	Converge & Diverge
Geometric		Converge: Diverge:
Divergent		Converge: Diverge:
Harmonic/Telescoping		Converge: Diverge:
Integral		Converge: Diverge:
P-Series		Converge: Diverge:
Comparison		Converge: Diverge:
Limit Comparison		Converge: Diverge:
Alternating		Converge: Diverge:
Ratio		Converge: Diverge:
Root		Converge: Diverge:



Key: 1-E 2-B 3-B 4-D 5-C 6-A 7-A 8-C 9-D 10-A 11-D 12-C 13-D 14-C 15-C 16-D 17-C 18-E

Taylor Polynomial-centered at $x = a$

$$\sum_{n=0}^{\infty} \frac{f^n(a)}{n!} (x-a)^n = f(x) = \frac{f(a)}{0!} (x-a)^0 + \frac{f'(a)}{1!} (x-a)^1 + \frac{f''(a)}{2!} (x-a)^2 + \frac{f^3(a)}{3!} (x-a)^3 + \frac{f^4(a)}{4!} (x-a)^4 + \dots$$

Maclaurin Polynomial-centered at $x = 0$

$$\sum_{n=0}^{\infty} \frac{f^n(0)}{n!} (x)^n = f(x) = \frac{f(0)}{0!} (x)^0 + \frac{f'(0)}{1!} (x)^1 + \frac{f''(0)}{2!} (x)^2 + \frac{f^3(0)}{3!} (x)^3 + \frac{f^4(0)}{4!} (x)^4 + \dots$$

Power Series-You must know

$f(x)$	$f(x)$ written as a summation	$f(x)$ written as a polynomial
$\frac{1}{1-x} =$	=	
$\frac{1}{1+x} =$	=	
$\sin(x) =$	=	
$\cos(x) =$	=	
$e^x =$	=	
$\ln(1+x) =$	=	

Error Bound: $\frac{k}{(n+1)!} |x-a|^{n+1}$ $k = |f^{n+1}(x)|$ or $|f^{n+1}(a)|$ the bigger of the the two or the one you can find.

Practice:

1. The series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{(-1)^n e^n}{n^2}$ diverges because,

- I. The terms do not tend to 0 as n tends to ∞ .
- II. The terms are not all positive.

III. $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| > 1$.

- a.) I only
- b.) II only
- c.) III only
- d.) I and II only
- e.) I and III only

2. The interval of convergence for the series

$$\sum_{n=1}^{\infty} \frac{(3x+2)^{n+1}}{n^{\frac{5}{2}}}$$

- a.) $-1 \leq x < -\frac{1}{3}$
- b.) $-1 < x \leq -\frac{1}{3}$
- c.) $-1 \leq x \leq -\frac{1}{3}$
- d.) $\frac{1}{3} \leq x \leq 1$
- e.) $-1 < x < \frac{1}{3}$

3. Given that $f(x) = \sum_{n=0}^{\infty} \frac{n(x-a)^n}{2^n}$ on the interval of convergence of the Taylor series, $f^{(4)}(a) =$

- a.) 0
- b.) 6
- c.) 9
- d.) $\frac{1}{4}$
- e.) $\frac{1}{4!}$

4. Which of the following converge?

I. $\sum_{n=1}^{\infty} \left(\frac{n^2 - n + 5}{n^2 + 1} \right)$

II. $\sum_{n=1}^{\infty} \frac{(-1)^n 3}{n}$

III. $\sum_{n=1}^{\infty} \left(\frac{\cos 2n\pi}{n^2} \right)$

- a.) I & II only
 b.) I & III only
 c.) II & III only
 d.) They all do.
 e.) None of them do

6. The Taylor series for $\ln(1+2x)$ about $x=0$ is

a.) $2x - \frac{(2x)^2}{2} + \frac{(2x)^3}{3} - \frac{(2x)^4}{4} + \dots$

b.) $2x - 2x^2 + 8x^3 - 16x^4 + \dots$

c.) $2x - 4x^2 + 16x^3 + \dots$

d.) $2x - x^2 + \frac{8x^3}{3} - 4x^4 + \dots$

e.) $2x - \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} - \frac{(2x)^4}{4!} + \dots$

8. The radius of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(x-3)^n (n+1)}{2^n (2n+1)}$$

- a.) 4
 b.) 3
 c.) 2
 d.) 1
 e.) 0

10. Which of the following series is convergent?

I. $\sum_{n=1}^{\infty} \frac{1}{n^3}$ II. $\sum_{n=1}^{\infty} \frac{2}{n+1}$ III. $\sum_{n=1}^{\infty} \frac{3^n}{n \cdot 2^n}$

- a.) I only
 b.) II only
 c.) III only
 d.) I and III only
 e.) I, II, & III

5. Which of the following series is divergent?

a.) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

b.) $\sum_{n=1}^{\infty} \frac{n+1}{n!}$

c.) $\sum_{n=2}^{\infty} \frac{1}{n \ln(n)}$

d.) $\sum_{n=1}^{\infty} \frac{\ln n}{2^n}$

e.) $\sum_{n=1}^{\infty} \frac{n}{2^n}$

7. The radius of convergence of $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} (x-2)^n}{n \cdot 3^n}$ is

- a.) 3
 b.) 2
 c.) 1
 d.) 0
 e.) ∞

9. What is the sum $\frac{5}{2} + \frac{5}{4} + \frac{5}{8} + \frac{5}{16} + \dots$?

- a.) 2
 b.) $\frac{75}{16}$
 c.) $\frac{315}{64}$
 d.) 5
 e.) The series diverges

11. What is the third-degree Taylor polynomial for $f(x) = \tan x$ at $x=0$?

- a.) $x - \frac{x^3}{3!}$
 b.) $x + \frac{x^3}{3!}$
 c.) $x - \frac{x^3}{3}$
 d.) $x + \frac{x^3}{3}$
 e.) $x + \frac{2x^3}{3}$

12. The Maclaurin series for the function f is

given by $f(x) = \sum_{n=0}^{\infty} \left(-\frac{x}{4}\right)^n$. What is the value of

$f(3)$?

- a.) -3
- b.) $-\frac{3}{7}$
- c.) $\frac{4}{7}$
- d.) $\frac{13}{16}$
- e.) 4

14. What is the radius of convergence of the

series $\sum_{n=0}^{\infty} \frac{(x-4)^{2n}}{3^n}$?

- a.) $2\sqrt{3}$
- b.) 3
- c.) $\sqrt{3}$
- d.) $\frac{\sqrt{3}}{2}$
- e.) 0

16. The power series $\sum_{n=0}^{\infty} a_n(x-3)^n$ converges at

$x=5$. Which of the following must be true?

- a.) The series diverges at $x=0$.
- b.) The series diverges at $x=1$.
- c.) The series converges at $x=1$.
- d.) The series converges at $x=2$.
- e.) The series converges at $x=6$.

13. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{8^n}{n!}$ II. $\sum_{n=1}^{\infty} \frac{n!}{n^{100}}$ III. $\sum_{n=1}^{\infty} \frac{n+1}{(n)(n+2)(n+3)}$

- a.) I only
- b.) II only
- c.) III only
- d.) I and III only
- e.) I, II, & III

15. For $x > 0$, the power series

$1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \dots + (-1)^n \frac{x^{2n}}{(2n+1)!} + \dots$ converges

to which of the following?

- a.) $\cos(x)$
- b.) $\sin(x)$
- c.) $\frac{\sin(x)}{x}$
- d.) $e^x - e^{x^2}$
- e.) $1 + e^x - e^{x^2}$

17. For what values of p will both series $\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$

and $\sum_{n=1}^{\infty} \left(\frac{p}{2}\right)^n$ converge?

- a.) $-2 < p < 2$ only
- b.) $-\frac{1}{2} < p < \frac{1}{2}$ only
- c.) $\frac{1}{2} < p < 2$ only
- d.) $p < \frac{1}{2}$ & $p > 2$
- e.) There are no such values of p .

18. Let f be a function having derivatives of all orders for $x > 0$ such that $f(3) = 2$, $f'(3) = -1$, $f''(3) = 6$, and $f'''(3) = 12$. Which of the following is the third-degree Taylor polynomial for f about $x = 3$?

- a.) $2 - x + 6x^2 + 12x^3$
- b.) $2 - x + 3x^2 + 2x^3$
- c.) $2 - (x-3) + 6(x-3)^2 + 12(x-3)^3$
- d.) $2 - (x-3) + 3(x-3)^2 + 4(x-3)^3$
- e.) $2 - (x-3) + 3(x-3)^2 + 2(x-3)^3$