BC Calculus — Name____________________Pd.____Seat #_____

Review #1 — Infinite Series Day 9

Determine whether each of the series converges or diverges. Identify which test you used.

1. $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by p-series $p=2>1$.

2. $\sum_{n=0}^{\infty} \frac{1}{2^n}$

3. $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ $\quad b_n = \frac{n}{n^2} = \frac{1}{n}$

$\lim_{n\to\infty} \frac{n}{n^2+1} \cdot \frac{n}{1} = 1$ & $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges by p-series $p=1 \leq 1$.

diverges by limit comparison

4. $\sum_{n=2}^{\infty} \frac{1}{\ln n}$

5. $\sum_{n=0}^{\infty} \left|\frac{n!}{2^n}\right|$

$R = \lim_{n\to\infty} \frac{(n+1)!}{2^{n+1}} \cdot \frac{2^n}{n!} = \lim_{n\to\infty} \frac{\cancel{2^n}(n+1)\cancel{n!}}{\cancel{2^n}\cdot 2^1 \cancel{n!}} = \infty > 1$

diverges by Ratio test $|R| = \infty > 1$.

6. $\sum_{n=1}^{\infty} \frac{\cos \pi n}{n}$

7. $\sum_{n=3}^{\infty} \frac{(-1)^n}{n \ln n - 1}$ $\quad a_n$ = alternating

& $\lim_{n\to\infty} \frac{1}{n \cdot \ln n - 1} = 0$

converges by alternating

8. $\sum_{n=1}^{\infty} \frac{n}{\ln(n)}$

9. $\sum_{n=1}^{\infty} \frac{n-1}{n^3+n+1}$ $\quad b_n = \frac{n}{n^3} = \frac{1}{n^2}$

$\lim_{n\to\infty} \frac{n-1}{n^3+n+1} \cdot \frac{n^2}{1} = 1 > 0$ & $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by p-series $p=2>1$.

10. $\sum_{n=1}^{\infty} \frac{(-1)^n \cdot (n+1) \cdot 3^n}{n!}$

11. $\sum_{n=5}^{\infty}\frac{3^n}{5^n} = \sum_{n=5}^{\infty}\left(\frac{3}{5}\right)^n$

converges by Ratio $|R| = \frac{3}{5} < 1$.

12. $\sum_{n=0}^{\infty}\frac{(-1)^n \cdot 2^n}{3^n}$

13. $\sum_{n=1}^{\infty}\frac{n}{n^3+1}$ $\quad \frac{1}{n^2} > \frac{n}{n^3+1}$

& $\sum_{n=1}^{\infty}\frac{1}{n^2}$ converges by p-series $p = 2 > 1$.

Converges by comparison

14. $\sum_{n=1}^{\infty}\frac{(-1)^n\sqrt{(n)}}{n+1}$

15. $\sum_{n=0}^{\infty}\left(\sqrt{3}\right)^n$

diverges by Ratio $|R| = \sqrt{3} \geq 1$.

16. $\sum_{n=0}^{\infty}\frac{n^2-n-5}{n^4+2n^3-3n+1}$

17. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^3}$ Alternating

& $\lim_{n\to\infty}\frac{1}{n^3} = 0$

Converges by alternating

18. $\sum_{n=1}^{\infty} ne^{-n^2}$

19. $\sum_{n=1}^{\infty}\frac{e^n-5}{e^n}$

$\lim_{n\to\infty}\frac{e^n}{e^n} - \frac{5}{e^n} = \lim_{n\to\infty} 1 - \frac{5}{e^n} = 1 \neq 0$

diverges by divergence test $\lim_{n\to\infty} = 1 \neq 0$.

20. $\sum_{n=1}^{\infty}\frac{7}{\sqrt[4]{n^5}}$

BC Calculus Name____________________Pd.____Seat #_____
Review #1 Infinite Series Day 9

Determine whether each the following series converges conditionally, converges absolutely, or diverges:

21. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

Positive series $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ diverges by p-series $p=\frac{1}{2} \leq 1$.

Alternating series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ is alternating & $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0 \therefore$ converges by alternating

Converges conditionally

22. $\sum_{n=2}^{\infty} \frac{(-1)^n \ln n}{n}$

23. $\sum_{n=2}^{\infty} \frac{(-1)^n n}{(\ln n)^2}$

Positive series $\sum_{n=2}^{\infty} \frac{n}{(\ln n)^2}$

$\frac{1}{n} < \frac{n}{(\ln(n))^2}$ & $\sum_{n=2}^{\infty} \frac{1}{n}$ diverges by p-series $p=1 \leq 1$.

$\therefore$ diverges by comparison

Alternating $\frac{(-1)^n n}{(\ln n)^2}$

let n=even

$\lim_{n\to\infty} \frac{n}{(\ln n)^2} \frac{\infty}{\infty}$

$\lim_{n\to\infty} \frac{1}{2(\ln n)^1 \cdot \frac{1}{n}}$

$\lim_{n\to\infty} \frac{n}{2(\ln n)} \frac{\infty}{\infty}$

$\lim_{n\to\infty} \frac{1}{2 \cdot \frac{1}{n}} = \infty$

let n=odd

$\lim_{n\to\infty} \frac{-n}{(\ln n)^2} \frac{\infty}{\infty}$

$\lim_{n\to\infty} \frac{-1}{2 \cdot (\ln n)^1 \cdot \frac{1}{n}}$

$\lim_{n\to\infty} \frac{-n}{2 \cdot \ln n} \frac{\infty}{\infty}$

$\lim_{n\to\infty} \frac{-1}{2 \cdot \frac{1}{n}} = -\infty$

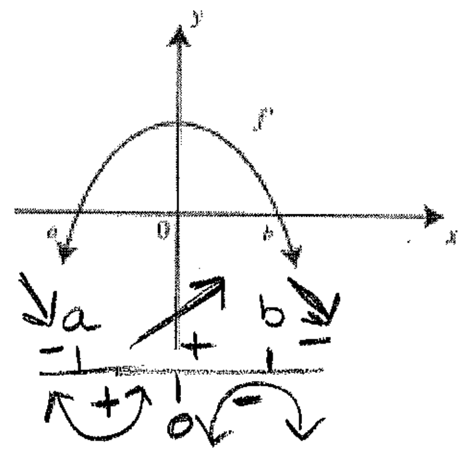
diverges by divergence lim = dne

diverges

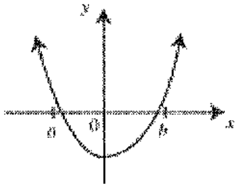
24. $\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{e^n - 1}$

**Review:**

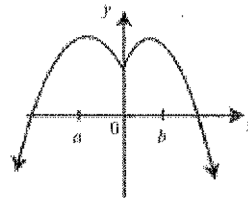
R1. The graph of f' is shown in the figure to the right. Which of the following is a possible graph of f?



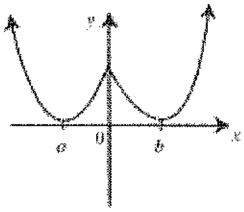
a.



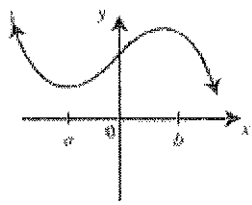
b.



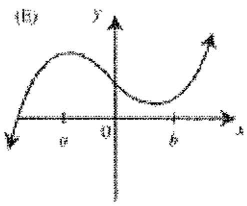
c.



d.



e.



R2. What is the $\lim_{h\to 0}\dfrac{\csc\left(\frac{\pi}{4}+h\right)-\csc\left(\frac{\pi}{4}\right)}{h} = f'\left(\frac{\pi}{4}\right)$

a. $\sqrt{2}$
b. $-\sqrt{2}$
c. 0
d. $-\dfrac{\sqrt{2}}{2}$
e. undefined

given $f(x) = \csc x$
$f'(x) = -\csc x \cot x$
$f'\left(\frac{\pi}{4}\right) = -\left(\frac{2}{\sqrt{2}}\right)(1)$
$= \frac{-2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\sqrt{2}$

R3. If a function is continuous for all values of x, which of the following statements is always true?

I. $2\int_a^b f(x)dx = \int_{2a}^{2b} f(x)dx$

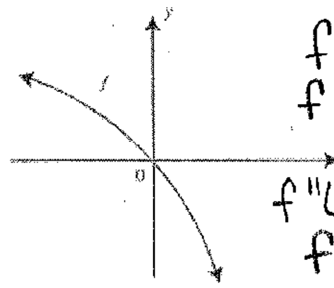
II. $\int_a^b f(x)dx = \int_b^a -f(x)dx$

III. $\left|\int_a^b f(x)dx\right| = \int_a^b |f(x)|dx$

a. I only b. I & II c. II only
d. II & III e. I, II, & III

R4. The graph of f is shown in the figure to the right. f is twice differentiable. Which of the following has the largest value: $f(0), f'(0), f''(0)$

a. f(0) b. f'(0) c. f''(0)
d. f(0) & f'(0) e. f'(0) & f''(0)



$f(0) = 0$
$f'(0) =$ neg b.c. $f(x)$ is decreasing
$f''(0) =$ neg b.c. $f(x)$ is concave down

**Answers:**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1. C | 2. C | 3. D | 4. D | 5. D | 6. C | 7. C | 8. D |
| 9. C | 10. C | 11. C | 12. C | 13. C | 14. C | 15. D | 16. C |
| 17. C | 18. C | 19. D | 20. C | 21. Conditional | | 22. Conditional | |
| 23. Diverge | | 24. Absolute | | R1. D | R2. B | R3. C | R4. A |