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Supplement: Absolute vs Conditional — Infinite Series Day 8

Test	What does it look Like?	Converge & Diverge
Geometric	$\sum_{n=1}^{\infty} C(R)^n$	Converge: $\|R\| < 1$ Diverge: $\|R\| \geq 1$
Divergent	$\sum_{n=1}^{\infty} a_n$	Converge: not this test Diverge: $\lim_{n\to\infty} a_n \neq 0$
Harmonic/Telescoping	$\sum_{n=1}^{\infty} a_n - a_{n+1}$	Converge: cancel & add terms = # Diverge: cancel & add terms = $+\infty$ or $-\infty$
Integral	$\sum_{n=1}^{\infty} a_n$	a_n = pos, dec, & continuous $[1,\infty)$ Converge: $\int a_n$ = # Diverge: $\int a_n = -\infty, \infty$, or dne
P-Series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	Converge: $p > 1$ Diverge: $p \leq 1$
Comparison	$\sum_{n=1}^{\infty} a_n$	Converge: big $(b_n) > a_n$ & $\sum b_n$ converges Diverge: small $(s_n) < a_n$ & $\sum s_n$ diverges
Alternating	$\sum_{n=1}^{\infty} (-1)^n a_n$	Converge: $\lim_{n\to\infty} a_n = 0$ Diverge: not this test
Ratio	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \left\| \frac{a_{n+1}}{a_n} \right\| = R$ Converge: $R < 1$ Diverge: $R > 1$
Conditional V.S. Absolute	Not a test. This is a type of convergence	Conditional Convergence Absolute Convergence

Show that each series is absolutely convergent, conditionally convergent, or divergent.

1. $\sum_{n=1}^{\infty} \frac{1}{(\ln 3)^n}$ ← All positive so could only be [1] absolute or [2] diverges

$$\sum_{n=1}^{\infty} \left(\frac{1}{\ln 3}\right)^n$$

converges by geometric

$|R| = \frac{1}{\ln 3} < 1.$

∴ Converges Absolutely

2. $\sum_{n=2}^{\infty} \frac{\ln n}{n}$ ← All positive so could only be [1] absolute or [2] diverges

3. $\sum_{n=1}^{\infty} \frac{5n^3 - 3n}{(n^3+n)(n^2+5)}$ ← All positive [1] Absolute [2] divergent

Reminder: Geometric & Ratio Are Absolute or diverge converge

Reminder: Ratio & geometric: Are absolute yes OR absolute no!

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4. $\sum_{n=1}^{\infty} n\sin\left(\frac{1}{n}\right)$ ($\infty (0)$)

All positive
1. Absolute
2. diverges

$\sum_{n=1}^{\infty} \frac{\frac{d}{dn}[\sin(\frac{1}{n})]}{\frac{d}{dn}[n^{-1}]} = \lim_{n\to\infty}$

$\lim_{n\to\infty} \frac{\cos(n^{-1})\cdot \cancel{-1\cdot n^{-2}}}{\cancel{-1\cdot n^{-2}}}$

$\lim_{n\to\infty} \cos\left(\frac{1}{n}\right) = \cos(0) = 1 \neq 0$

∴ diverges by divergence

diverges

5. $\sum_{n=0}^{\infty} \frac{e^n}{1+e^{2n}}$

All positive
1. Absolute
2. diverges

6. $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$

All positive
1. Absolute
2. diverges

7. $\sum_{n=1}^{\infty} \frac{3^{n-1}+1}{3^n}$

All positive
1. Absolute
2. diverges

8. $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\ln n}$

Alternating
1. Absolute?
2. Conditional?
3. diverges

Look at Positive $\sum$

$\sum_{n=2}^{\infty} \frac{1}{\ln n}$ $\quad \frac{1}{n} < \frac{1}{\ln n}$ & $\sum \frac{1}{n}$ diverges by p-series $p = 1 \leq 1$.

∴ diverges by comparison

$\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\ln n}$ is alternating & $\lim_{n\to\infty} \frac{1}{\ln n} = 0$

∴ converges by Alternating

Converges Conditionally

9. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}10^n}{n^{10}}$

Alternating
1. Absolute?
2. Conditional?
3. diverges?

10. $\sum_{n=1}^{\infty} (-1)^{n+1}\frac{\sqrt{n}+1}{n+1}$

Alternating
1. Absolute
2. Conditional
3. diverges

11. $\sum_{n=2}^{\infty} (-1)^{n+1}\frac{\ln n}{\ln n^2}$

Alternating
1. Absolute
2. Conditional
3. diverges

Look at Positive

$\sum_{n=2}^{\infty} \frac{\ln n}{\ln n^2}$ $\frac{\infty}{\infty}$

$\lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{n^2}\cdot 2n} = \frac{\frac{1}{\cancel{n}}}{\frac{2\cancel{n}}{n^{\cancel{2}}}} = \frac{1}{2} \neq 0$

∴ Diverges by divergence

Alternating $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}\ln n}{\ln n^2}$

let n = even $\lim_{n\to\infty} \frac{-\ln(n)}{\ln(n^2)} = -\frac{1}{2}$

let n = odd $\lim_{n\to\infty} \frac{\ln(n)}{\ln(n^2)} = \frac{1}{2}$

$\lim_{n\to\infty} \frac{(-1)^{n+1}\ln(n)}{\ln(n^2)} = \text{dne} \neq 0$ ∴ diverges by divergence

diverges

12. $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n^2}\right)$

All positive
1. Absolute
2. diverges

Reminder: Ratio & geometric tests: are absolute yes or absolute no!

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13. $\sum_{n=1}^{\infty}(-1)^{n+1}(0.1)^n$ Alternating
[1] Absolute
[2] Conditional
[3] diverges

14. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n+1}{n^2}$ Alternating
[1] Absolute
[2] Conditional
[3] diverges

15. $\sum_{n=1}^{\infty}\left|(-1)^n n^2\left(\frac{2}{3}\right)^n\right|$ Alternating
[1] Absolute
[2] Conditional
[3] diverges

$R=\lim_{n\to\infty}\frac{(n+1)^2\cdot 2^{n+1}}{3^{n+1}}\cdot\frac{3^n}{2^n\cdot n^2}$

$R=\lim_{n\to\infty}\frac{\cancel{2^n}\cdot 2^1\cdot\cancel{3^n}(n+1)^2}{\cancel{2^n}\cdot\cancel{3^n}\cdot 3^1(n^2)}=\frac{2}{3}$

converges by Ratio $|R|=\frac{2}{3}<1$

Converges absolutely

16. $\sum_{n=2}^{\infty}\frac{(-1)^{n+1}}{n\ln n}$ Alternating
[1] Absolute
[2] Conditional
[3] diverges

17. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n!}{2^n}$ Alternating
[1] Absolute
[2] Conditional
[3] diverges

18. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin n}{n^2}$ Alternating
[1] Absolute
[2] Conditional
[3] diverges

Look at Positive

$\sum_{n=1}^{\infty}\frac{\sin(n)}{n^2}$ & $\sum_{n=1}^{\infty}\frac{1}{n^2}$ converges by p-series $p=2>1$.

$\frac{1}{n^2}>\frac{\sin(n)}{n^2}$

$\therefore \sum_{n=1}^{\infty}\frac{\sin(n)}{n^2}$ converges by comparison

Converges absolutely

If the positive series converges then the Alternating must too.

19. $\sum_{n=1}^{\infty}\frac{(-1)^n}{1+\sqrt{n}}$ Alternating
[1] Absolute
[2] Conditional
[3] diverges

Look at Positive

$\sum_{n=1}^{\infty}\frac{1}{1+\sqrt{n}}$ $b_n=\frac{1}{\sqrt{n}}$

$\lim_{n\to\infty}\frac{1}{1+\sqrt{n}}\cdot\frac{\sqrt{n}}{1}=1>0$ & $\sum\frac{1}{n^{1/2}}$ diverges p-series $p=1/2<1$

$\therefore\sum_{n=1}^{\infty}\frac{1}{1+\sqrt{n}}$ diverges by limit comparison

Alternating: $\sum_{n=1}^{\infty}\frac{(-1)^n}{1+\sqrt{n}}$

$a_n=\frac{(-1)^n}{1+\sqrt{n}}$ is alternating & $\lim_{n\to\infty}a_n=0$ $\therefore$ converges by Alternating

Converges conditionally

20. $\sum_{n=1}^{\infty}\frac{\cos n\pi}{n\sqrt{n}}$ Alternating
[1] Absolute
[2] Conditional
[3] diverges

21. $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}+\sqrt{n+1}}$ Alternating
[1] Absolute
[2] Conditional
[3] diverges

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Review:

R1. $\int_0^8 x^{\frac{2}{3}}dx$ $\frac{3}{5}x^{5/3}\Big|_0^8$

a. $\frac{1}{3}$

b. $\frac{96}{5}$

c. $\frac{4}{3}$

d. $-\frac{1}{3}$

e. $-\frac{96}{5}$

$\frac{3}{5}\left[(\sqrt[3]{8})^5-(\sqrt[3]{0})^5\right]$

$\frac{3}{5}(2)^5$

$\frac{3\cdot 32}{5}=\frac{96}{5}$

R2. $\lim_{x\to\infty}\frac{x^2+4x-5}{x^3-1}$ = end behav.oR = 0

a. 0

b. $\frac{1}{3}$

c. 5

d. $-\infty$

e. ∞

R3. What is the $\lim_{x\to-2} f(x)$, if

$f(x)=\begin{cases}|x-1| & \text{if } x>-2 \\ 2x+7 & \text{if } x\le -2\end{cases}$ Right / left

a. −3

b. 1

c. 3

d. 11

e. nonexistent

$\lim_{x\to-2^+}|-2-1|=|-3|=3$

$\lim_{x\to-2^-} 2(-2)+7=-4+7=3$

If $\lim_{x\to-2^+}=\lim_{x\to-2^-}$ Then $\lim_{x\to-2}=3$

$3=3$

R4. $\int_{\frac{\pi}{2}}^{x} 2\cos(t)dt =$

a. $2\cos x$

b. $-2\cos x$

c. $2\sin x$

d. $-2\sin x+2$

e. 2sinx-2

$2\sin(t)\Big|_{\frac{\pi}{2}}^{x}$

$2\sin(x)-2\sin\left(\frac{\pi}{2}\right)$

$2\sin(x)-2(1)$

$2\sin x-2$

Answers:

1. Converges absolutely
2. Diverges
3. Converges absolutely
4. Diverges
5. Converges absolutely
6. Converges absolutely
7. Diverges
8. Conditionally converges
9. Diverges
10. Conditionally converges
11. Diverges
12. Diverges
13. Converges absolutely
14. Conditionally converges
15. Converges absolutely
16. Conditionally converges
17. Diverges
18. Converges absolutely
19. Conditionally converges
20. Converges absolutely
21. Conditionally converges

R1. B R2. A R3. C R4. E