BC Calculus
Supplement: All tests but Root

Name Homework Guide Pd.____
Infinite Series Day 7

Test	What does it look Like?	Converge & Diverge
Geometric	$\sum_{n=1}^{\infty} c(R)^n$	Converge: $\lvert R\rvert < 1$ Diverge: $\lvert R\rvert \geq 1$
Divergent	$\sum_{n=1}^{\infty} a_n$	Converge: Not this Test! If: $\lim_{n\to 0} a_n = 0$ Then: Try new test Diverge: $\lim_{n\to\infty} a_n \neq 0$
Harmonic/Telescoping	$\sum_{n=1}^{\infty} a_n - a_{n+1}$	Converge: Cancel & add terms = # Diverge: Cancel & add terms = ∞ or $-\infty$
Integral	$\sum_{n=1}^{\infty} a_n$	a_n: is positive, decreasing, & continuous $[0,\infty)$ Converge: $\int a_n =$ some # Diverge: $\int a_n = -\infty$ or ∞ or dne
P-Series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	Converge: $p > 1$ Diverge: $p \leq 1$
Comparison	$\sum_{n=1}^{\infty} a_n$	Converge: bigger $(b_n) > a_n$ & $\sum b_n$ converges Diverge: small $(s_n) < a_n$ & $\sum s_n$ diverges
Limit Comparison	$\sum_{n=1}^{\infty} a_n$	Converge: $\lim_{n\to\infty} \frac{a_n}{b_n} > 0$ & $\sum b_n$ converges Diverge: $\lim_{n\to\infty} \frac{a_n}{b_n} > 0$ & $\sum b_n$ diverges
Alternating	$\sum_{n=1}^{\infty} (-1)^n a_n$	Converge: $\lim_{n\to\infty} a_n = 0$ Diverge: Not this test! If $\lim_{n\to\infty} a_n \neq 0$ Then divergence test
Ratio	$\sum_{n=1}^{\infty} a_n$	$R = \lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert$ Converge: $R < 1$ Diverge: $R > 1$ Inconclusive: $R = 1$
Root	$\sum_{n=1}^{\infty} (a_n)^n$	$L = \lim_{n\to\infty} \sqrt[n]{a_n}$ Converge: $L < 1$ Diverge: $L > 1$ Inconclusive: $L = 1$

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1. $\sum_{n=1}^{\infty} \frac{2^n}{n^3}$

2. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+2n}}$

$\frac{1}{\sqrt{n^3}} = \frac{1}{n^{3/2}} > \frac{1}{\sqrt{n^3+2n}}$

& $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ converges by p-series $p = 3/2 > 1$.

$\therefore \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+2n}}$ converges by comparison

3. $\sum_{n=1}^{\infty} \frac{n}{e^{n^2}}$

4. $\sum_{n=1}^{\infty} (-1)^n \frac{n}{\ln n}$

let n = even

$\lim_{n\to\infty} \frac{\frac{d}{dn}[n}{\frac{d}{dn}[\ln n} \frac{\infty}{\infty}$

$\lim_{n\to\infty} \frac{1}{\frac{1}{n}}$

$\lim_{n\to\infty} n = \infty$

let n = odd

$\lim_{n\to\infty} \frac{-n}{\ln n} \frac{\infty}{\infty}$

$\lim_{n\to\infty} \frac{-1}{\frac{1}{n}}$

$\lim_{n\to\infty} -n = -\infty$

diverges by divergence b.c $\lim_{n\to\infty} \frac{(-1)^n n}{\ln n} = \text{dne} \neq 0$.

5. $\sum_{n=1}^{\infty} \left(\frac{1}{n^2} - \frac{1}{n}\right)$

$\sum_{n=1}^{\infty} \frac{1-n}{n^2}$ $\quad b_n = \frac{-1}{n}$

$\lim_{n\to\infty} \frac{1-n}{n^2} \cdot \frac{n}{-1} = 1 > 0$ & $\sum_{n=1}^{\infty} \frac{-1}{n}$ diverges by p-series $p = 1 \leq 1$.

$\therefore \sum_{n=1}^{\infty} \frac{1}{n^2} - \frac{1}{n}$ diverges by limit comparison

Answers:

1. D:ratio 2. C:comparison 3. C:integral 4. D:divergence 5. D: limit comparison

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6. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n^3}}$

7. $\sum_{n=1}^{\infty} \frac{n+1}{n(n+2)}$

8. $\sum_{n=1}^{\infty} \frac{n!}{e^n}$

$R = \lim_{n\to\infty} \frac{(n+1)!}{e^{n+1}} \cdot \frac{e^n}{n!} = \frac{\cancel{e^n}(n+1)\cancel{n!}}{\cancel{e^n}\cdot e^1 \cdot \cancel{n!}}$

$R = \lim_{n\to\infty} \frac{n+1}{e} = \infty > 1$

$\therefore \sum_{n=1}^{\infty} \frac{n!}{e^n}$ diverges by Ratio $|R| = \infty > 1$.

9. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{4^n n}$

10. $\sum_{n=1}^{\infty} \frac{1}{2^n n!}$

11. $\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n$

Answers

6. D:p
7. D:limit comparion
8. D:ratio
9. C:alternating
10. C:ratio
11. C:geometric

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12. $\sum_{n=0}^{\infty} \frac{2^n}{3^n n!}$

$\frac{2^n}{3^n} > \frac{2^n}{3^n \cdot n!}$

$\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n$ converges by geometric $|R| = \frac{2}{3} < 1$.

$\therefore \sum_{n=0}^{\infty} \frac{2^n}{3^n n!}$ converges by comparison

13. $\sum_{n=0}^{\infty} \frac{1}{n^2+1}$

14. $\sum_{n=1}^{\infty} \frac{5n-3}{n^2-2n+5}$

15. $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

16. $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}}$

$u = n^2+1$
$du = 2n\,dn \quad \frac{1}{2}du = n\,dn$

$\lim_{R\to\infty} \int_1^R \frac{n}{\sqrt{n^2+1}}\,dn$

$\lim_{R\to\infty} \frac{1}{2}\int_1^R u^{-1/2}\,du$

$\lim_{R\to\infty} \frac{1}{2} 2 u^{1/2} \Big|_1^R$

$\lim_{R\to\infty} \sqrt{n^2+1}\ \Big|_1^R$

$\lim_{R\to\infty} \sqrt{R^2+1} - \sqrt{2}$

$\infty - \sqrt{2}$

$\therefore \sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}}$ diverges by Integral b.c. a_n = pos, dec, & conti & $\int a_n = \infty$

17. $\sum_{n=1}^{\infty} \frac{n2^n}{4n^3+1}$

18. $\sum_{n=1}^{\infty} \frac{e^{2n}}{n^n}$

$\sum_{n=1}^{\infty} \frac{(e^2)^n}{n^n} = \sum_{n=1}^{\infty} \left(\frac{e^2}{n}\right)^n \quad \lim_{n\to\infty} \sqrt[n]{\left(\frac{e^2}{n}\right)^n} = \lim_{n\to\infty} \frac{e^2}{n} = 0 < 1$

$\therefore \sum_{n=1}^{\infty} \frac{e^{2n}}{n^n}$ converges by Root $L = 0 < 1$

Answers:

12. C:comparison
13. C:comparison
14. D:limit comparison
15. D: limit comparison
16. D:integral
17. D:ratio
18. C:~~ratio~~ Root