Homework

BC Calculus

Comparison Tests

Name Guide Pd.____

Infinite Series Day 5

1.Suppose $\sum a_n$ and $\sum b_n$ are series with positive terms and $\sum b_n$ is known to be convergent.

A.)If $a_n > b_n$ for all n, what can you say about $\sum a_n$? Why? Nothing

B.) If $a_n < b_n$ for all n, what can you say about $\sum a_n$? Why? $b_n > a_n$ & $\sum a_n$ converges then $\sum a_n$ converges by comparison

2.Suppose $\sum a_n$ and $\sum b_n$ are series with positive terms and $\sum b_n$ is known to be divergent.

A.)If $a_n > b_n$ for all n, what can you say about $\sum a_n$? Why?

B.) If $a_n < b_n$ for all n, what can you say about $\sum a_n$? Why?

Determine whether the series converges or diverges.

3. $\sum_{n=1}^{\infty} \frac{n}{2n^3+1}$

$\frac{n}{n^3} = \frac{1}{n^2}$

$\frac{1}{n^2} > \frac{n}{2n^3+1}$ & $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by p-series $p = 2 > 1$.

$\therefore \sum_{n=1}^{\infty} \frac{n}{2n^3+1}$ converges by comparison

4. $\sum_{n=2}^{\infty} \frac{n^3}{n^4-1}$

5. $\sum_{n=1}^{\infty} \frac{\cancel{n}}{\cancel{n}\sqrt{n}}$

$\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ diverges by p-series $p = 1/2 \le 1$.

6. $\sum_{n=1}^{\infty} \frac{n-1}{n^2\sqrt{n}}$

7. $\sum_{n=1}^{\infty} \frac{9^n}{3+10^n}$

$\left(\frac{9}{10}\right)^n > \frac{9^n}{3+10^n}$ & $\sum_{n=1}^{\infty} \left(\frac{9}{10}\right)^n$ converges by geometric $|R| = \frac{9}{10} < 1$

$\therefore \sum_{n=1}^{\infty} \frac{9^n}{3+10^n}$ converges by comparison

8. $\sum_{n=1}^{\infty} \frac{6^n}{5^n-1}$

BC Calculus
Comparison Tests

Name______________________Pd.____
Infinite Series Day 5

9. $\sum_{k=1}^{\infty} \frac{\ln k}{k}$

$\frac{1}{K} < \frac{\ln K}{K}$ & $\sum_{K=1}^{\infty} \frac{1}{K}$ diverges by p-series $p=1 \leq 1$.

$\therefore \sum_{K=1}^{\infty}$ diverges by Comparison

10. $\sum_{k=1}^{\infty} \frac{k \sin^2 k}{1+k^3}$

11. $\sum_{k=1}^{\infty} \frac{\sqrt[3]{k}}{\sqrt{k^3+4k+3}}$

12. $\sum_{k=1}^{\infty} \frac{(2k-1)(k^2-1)}{(k+1)(k^2+4)^2}$ $\frac{2K^3}{K^5} = b_n = \frac{2}{K^2}$

$\lim_{n\to\infty} \frac{(2K-1)(K^2-1)}{(K+1)(K^2+4)^2} \cdot \frac{K^2}{2} = 1 > 0$

& $\sum_{n=1}^{\infty} \frac{2}{K^2}$ converges by p-series $p=2>1$.

$\therefore \sum_{K=1}^{\infty} \frac{(2K-1)(K^2-1)}{(K+1)(K^2+4)^2}$ converges by limit comparison

13. $\sum_{n=1}^{\infty} \frac{\arctan n}{n^{1.2}}$

14. $\sum_{n=2}^{\infty} \frac{\sqrt{n}}{n-1}$ $\frac{\sqrt{n}}{n} = \frac{n^{1/2}}{n^1} = \frac{1}{n^{1/2}} = b_n$

$\lim_{n\to\infty} \frac{\sqrt{n}}{n-1} \cdot \frac{\sqrt{n}}{1} = 1 > 0$ & $\sum_{n=2}^{\infty} \frac{1}{n^{1/2}}$ diverges by p-series $p=\frac{1}{2} \leq 1$.

$\therefore \sum_{n=2}^{\infty} \frac{\sqrt{n}}{n-1}$ diverges by limit comparison

15. $\sum_{n=1}^{\infty} \frac{4^{n+1}}{3^n-2}$

16. $\sum_{n=2}^{\infty} \frac{1}{\sqrt[3]{3n^4+1}}$

$\frac{1}{n^{4/3}} > \frac{1}{\sqrt[3]{3n^4+1}}$ & $\sum_{n=2}^{\infty} \frac{1}{n^{4/3}}$ converges by p-series $p=4/3>1$.

$\therefore \sum_{n=2}^{\infty} \frac{1}{\sqrt[3]{3n^4+1}}$ converges by comparison test

BC Calculus
Comparison Tests

Name______________________Pd.____
Infinite Series Day 5

Review

R1. $\int_0^{\ln 2} e^{2x}dx =$

a. $\frac{3}{2}$
b. 3
c. 4
d. $e^2 - \frac{1}{2}$
e. $2e^2 - 1$

$\frac{e^{2x}}{2}\Big|_0^{\ln 2}$

$\frac{e^{2\cdot\ln 2}}{2} - \frac{e^0}{2}$

$\frac{e^{\ln 4}}{2} - \frac{1}{2} = \frac{4}{2} - \frac{1}{2} = \frac{3}{2}$

R2. The area of the region enclosed by the graph of $y = \sqrt{9 - x^2}$ and the x-axis is

a. 36
b. $\frac{9\pi}{2}$
c. 9π
d. 18π
e. 36π

$A = \int_{-3}^{3} \sqrt{9-x^2}\,dx$

OR

$\frac{1}{2}\pi(3)^2 = \frac{9\pi}{2}$

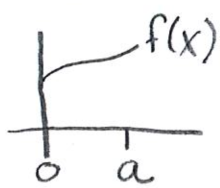
R3. If a function f is continuous for all real values of x, and a>0 and b<0, which of the following integrals always has the same value?

$I. \int_0^a f(x)dx \quad II. \int_b^{a+b} f(x-b)dx \quad III. \int_b^{a+b} f(x+b)dx$

Right b

left b

a. I & II only
b. I & III only
c. II & III only
d. I, II, & III
e. none



R4. What is the average value of the function $y = 2\sin(2x)$ on the interval $\left[0, \frac{\pi}{6}\right]$?

a. $-\frac{3}{\pi}$
b. $\frac{1}{2}$
c. $\frac{3}{\pi}$
d. $\frac{3}{2\pi}$
e. 6π

$\frac{1}{\frac{\pi}{6} - 0} \int_0^{\frac{\pi}{6}} 2\cdot\sin(2x)\,dx$

$\frac{6}{\pi}\left[\cancel{2}\frac{(-\cos(2x))}{2}\right]_0^{\frac{\pi}{6}}$

$\frac{6}{\pi}\left[-\cos\frac{\pi}{3} + \cos(0)\right]$

$\frac{6}{\pi}\left[-\frac{1}{2} + 1\right]$

$\frac{6}{\pi}\left[\frac{1}{2}\right] = \frac{3}{\pi}$

Answers:

1. A.) Nothing B.) Then $\sum_{n=1}^{\infty} a_n$ converges by the comparison test
2. A.)Then $\sum_{n=1}^{\infty} a_n$ is divergent by the comparison test. B.) Nothing

All of these you must tell A.) Converges or Diverges and use an appropriate test.

3. Converges	10. Converges	R1. A
4. Diverges	11. Converges	R2. B
5. Diverges	12. Converges	R3. A
6. Converges	13. Converges	R4. C
7. Converges	14. Diverges	
8. Diverges	15. Diverges	
9. Diverges	16. Converges	