BC Calculus Name_____________________Pd.____

Supplement: Geometric, Divergent, Integral, P-Series, & Comparison Infinite Series Day 4

| Test | What does it look Like? | Converge & Diverge |
|---|---|---|
| Geometric | $\sum_{n=0}^{\infty} C(R)^n$ | Converge: $\lvert R\rvert < 1$<br>Diverge: $\lvert R\rvert \geq 1$ |
| Divergent | $\sum_{n=0}^{\infty} a_n$ | Converge: not this test. If $\lim_{n\to\infty} a_n = 0$ Try another<br>Diverge: $\lim_{n\to\infty} a_n \neq 0$ |
| Harmonic/Telescoping | $\sum_{n=0}^{\infty} a_n - a_{n+1}$ | Converge: term = some #<br>Diverge: terms $= \infty$ OR $-\infty$ |
| Integral | $\sum_{n=0}^{\infty} a_n$ | $a_n$ must be continuous, decreasing, & positive<br>Converge: $\int a_n = \#$<br>Diverge: $\int a_n = \pm\infty$ |
| P-Series | $\sum_{n=0}^{\infty} \frac{1}{n^p}$ | Converge: $p > 1$<br>Diverge: $p \leq 1$ |
| Comparison | $\sum_{n=0}^{\infty} a_n$ | Converge: bigger series converges<br>Diverge: smaller series diverges |

Determine which series are convergent and which are divergent. State your reasoning.

1. $\sum_{n=1}^{\infty} \frac{1}{10^n}$ Geometric

2. $\sum_{n=1}^{\infty} \left(\frac{3}{8}\right)^{1-n}$ Geometric

$$\sum_{n=1}^{\infty} \left(\frac{3}{8}\right)^1 \left(\frac{3}{8}\right)^{-n}$$

$$\sum_{n=1}^{\infty} \frac{3}{8} \cdot \frac{3^{-n}}{8^{-n}}$$

$$\sum_{n=1}^{\infty} \frac{3}{8} \left(\frac{8}{3}\right)^n$$

diverges by geometric

$\lvert R\rvert = \frac{8}{3} \geq 1$

3. $\sum_{n=1}^{\infty} \frac{n}{n+2}$ Divergence Integral Comparison

4. $\sum_{n=1}^{\infty} \frac{5}{n}$ P-series

$$5 \sum_{n=1}^{\infty} \frac{1}{n^1}$$

diverges by p-series

$p = 1 \leq 1.$

5. $\sum_{n=1}^{\infty} \frac{\sin^2 n}{2^n}$ Comparison

6. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$ P-series

$$\sum_{n=1}^{\infty} \frac{1}{n^1 \cdot n^{1/2}}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$$

converges by p-series

$p = \frac{3}{2} > 1.$

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Determine which series are convergent and which are divergent. State your reasoning.

7. $\sum_{n=2}^{\infty} \frac{\ln n}{n}$ comparison Integral

8. $\sum_{n=2}^{\infty} \frac{\sqrt{n}}{\ln n}$ $\frac{\infty}{\infty}$ divergence

$\lim_{n\to\infty} \frac{\frac{d}{dn}[n^{1/2}]}{\frac{d}{dn}[\ln n]} = \lim_{n\to\infty} \frac{\frac{1}{2}n^{-1/2}}{1/n}$

$\lim_{n\to\infty} \frac{\frac{1}{2n^{1/2}} \cdot \frac{n}{1}}{\frac{1}{n} \cdot \frac{n}{1}} = \lim_{n\to\infty} \frac{n^{1/2}}{2} = \infty$

diverges by divergence test b.c. $\lim_{n\to\infty} a_n = \infty \neq 0$.

9. $\sum_{n=1}^{\infty} \frac{1+\cos n}{n^2}$ comparison

10. $\sum_{n=0}^{\infty} \frac{-2}{n+1}$ comparison

$\frac{-2}{n} < \frac{-2}{n+1}$ & $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges by p-series $p=1\leq 1$.

diverges by comparison b.c. $-\frac{2}{n} < \frac{-2}{n+1}$ & $\sum_{n=1}^{\infty} \frac{-2}{n}$ diverges

11. $\sum_{n=1}^{\infty} \frac{2^n}{3^n}$ geometric

12. $\sum_{n=1}^{\infty} \frac{1}{1+\ln n}$ comparison

$\frac{1}{n} < \frac{1}{\ln n+1}$ & $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges by p-series $p=1\leq 1$.

$\sum_{n=1}^{\infty} \frac{1}{1+\ln}$ diverges by comparison

13. $\sum_{n=1}^{\infty} \frac{1}{2n-1}$ comparison Integral

14. $\sum_{n=1}^{\infty} \frac{2^n}{n+1}$ $\frac{\infty}{\infty}$ divergence

$\lim_{n\to\infty} \frac{\frac{d}{dn}[2^n]}{\frac{d}{dn}[n+1]}$

$\lim_{n\to\infty} \frac{2^n \cdot \ln 2}{1} = \infty$

diverges by divergence test b.c. $\lim_{n\to\infty} a_n = \infty \neq 0$.

15. $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ comparison Integral

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Determine which series are convergent and which are divergent. State your reasoning.

16. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$ p-series

17. $\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$ Integral

18. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+2}}$ comparison

$\frac{1}{\sqrt{n^3}} > \frac{1}{\sqrt{n^3+1}}$

$\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ converges by p-series $p = \frac{3}{2} > 1$.

$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+2}}$ converges by comparison

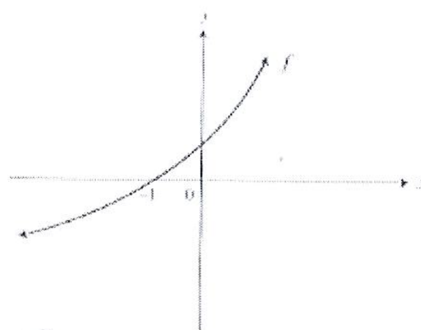
19. $\sum_{n=1}^{\infty} \frac{1}{(\ln 2)^n}$ geometric

20. $\sum_{n=1}^{\infty} \frac{1}{3^{n-1}+1}$ comparison

21. $\sum_{n=1}^{\infty} \frac{2x+3}{x-4}$ divergence

Review

R1. The graph of f is shown in the figure to the right and f is twice differentiable. Which of the following has the smallest value?



I. $f(-1)$

II. $f'(-1)$

III. $f''(-1)$

a. I b. II c. III

d. I & II e. II & III

$f(x)$ = increasing $\therefore f'(1) = +$

Smallest $\longrightarrow f(-1) = 0$

$f(x)$ = concave up $\therefore f''(1) = +$

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**Review continued**

R2. If $\frac{dy}{dx} = 3e^{2x}$, and at x=0, $y = \frac{5}{2}$, as solution to the differential equation is

a. $3e^{2x} - \frac{1}{2}$  b. $3e^{2x} + \frac{1}{2}$  c. $\frac{3}{2}e^{2x} + 1$

d. $\frac{3}{2}e^{2x} + 2$  e. $\frac{3}{2}e^{2x} + 5$

$\int dy = \int 3e^{2x}dx$  $\frac{5}{2} = \frac{3}{2}e^{0} + C$

$y = \frac{3e^{2x}}{2} + C$  $C = 1$  $y = \frac{3}{2}e^{2x} + 1$

R3. The position function of a moving particle is $s(t) = \frac{t^3}{6} - \frac{t^2}{2} + t - 3$ for $0 \le t \le 4$. What is the maximum velocity of the particle on the interval $0 \le t \le 4$? $V(1) = \frac{1}{2}(1)^2 - \cancel{1} + \cancel{1} = \frac{1}{2}$

a. $\frac{1}{2}$  b. 1  c. $\frac{14}{6}$  d. 4  e. 5

$v(t) = \frac{3}{6}t^2 - \frac{1}{2}(2t) + 1 = \frac{1}{2}t^2 - t + 1$

$v'(t)$ to find max  $v'(t) = t - 1$

$0 = t - 1$ $\boxed{t=1}$ when $v(1) = 1/2$

$t = 0$  $v(0) = 1$

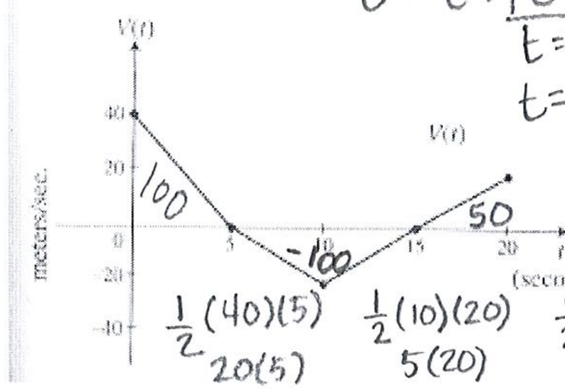
$t = 4$  $v(4) = \frac{1}{2}(16) - 4 + 1$

$8 - 4 + 1$

$5$

R4. The graph of the velocity functions of a moving particle is shown in the figure to the right. What is the total displacement of the particle during $0 \le t \le 20$?

a. 20m
b. 50m
c. 100m
d. 250m
e. 500m

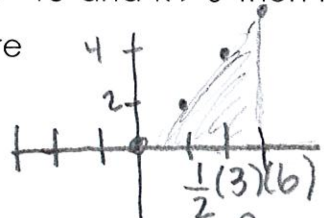



$\frac{1}{2}(40)(5)$  $\frac{1}{2}(10)(20)$  $\frac{1}{2}(5)(20)$

$20(5)$  $5(20)$  $5(10)$

R5. If $\int_{-k}^{k} |2x|\,dx = 18$ and $k > 0$ then the value(s) of k are

a. $-3$
b. $-3\sqrt{2}$
c. 3
d. $3\sqrt{2}$
e. 9




$\frac{1}{2}(3)(6)$

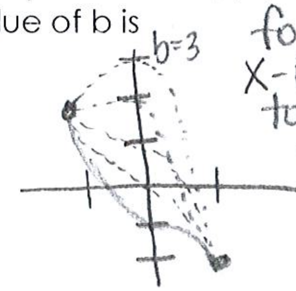
9

R6. A function f is continuous on [-1,1] and some of the values of are shown below:

| x | -1 | 0 | 1 |
|---|---|---|---|
| f(x) | 2 | B | -2 |

If f(x)=0 has only one solution, r, and r<0, then a possible value of b is

~~a. 3~~
~~b. 2~~
~~c. 1~~
~~d. 0~~
e. -1




for the x-intercept to be less than 0

b has to be -1

**Answers:**

| | | | |
|---|---|---|---|
| 1. Converges | 2. Diverges | 3. Diverges | 4. Diverges |
| 5. Converges | 6. Converges | 7. Diverges | 8. Diverges |
| 9. Converges | 10. Diverges | 11. Converges | 12. Diverges |
| 13. Diverges | 14. Diverges | 15. Diverges | 16. Converges |
| 17. Diverges | 18. Converges | 19. Diverges | 20. Converges |
| 21. Diverges | | | |

R1. A  R2. C  R3. E  R4. B  R5. C  R6. E