BC Calculus | Name____________________Pd.____

Supplement: Integral, P-Series, & Comparison | Infinite Series Day 3

Use the **integral test** to determine whether the series is convergent or divergent.

1. $\sum_{n=25}^{\infty} \frac{n^2}{(n^3+9)^{\frac{5}{2}}}$

$u = n^3+9$
$du = 3n^2\,dn$
$\frac{1}{3}du = n^2\,dn$

a_n is pos, dec, & continuous & $\int a'$

$\int_{25}^{\infty} \frac{n^2}{(n^3+9)^{5/2}}\,du$

$\lim_{R\to\infty} \frac{1}{3}\int_{25}^{R} u^{-5/2}\,du$

$\lim_{R\to\infty} \frac{1}{3}\cdot\frac{-2}{3}u^{-3/2}\Big|_{25}^{R}$

$\lim_{R\to\infty} -\frac{2}{9}(n^3+9)^{-3/2}\Big|_{25}^{R}$

$\lim_{R\to\infty} \frac{-2}{9(n^3+9)^{3/2}}\Big|_{25}^{R}$

$\lim_{R\to\infty} \frac{-2}{9(R^3+9)^{3/2}} \to 0 + \frac{2}{9(25^3+9)^{3/2}}$

$= \frac{2}{9(25^3+9)^{3/2}}$ $\therefore$ converges by integral

2. $\sum_{n=1}^{\infty} ne^{-n^2}$

Use the **p-series test** to determine whether the series is convergent or divergent.

3. $\sum_{n=1}^{\infty} n^{-\frac{1}{3}} = \sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$

diverges by p-series $p = \frac{1}{3} \leq 1$

4. $\sum_{n=1}^{\infty} \frac{1}{n^4}$

Use the **comparison test** to determine whether the series is convergent or divergent.

5. $\sum_{n=4}^{\infty} \frac{1}{n-3}$

diverges by comparison test b.c. $\frac{1}{n} < \frac{1}{n-3}$ & $\sum \frac{1}{n}$ diverges by p-series $p = 1 \leq 1$.

6. $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

7.-16. Use the geometric, divergent, integral, p-series, or comparison test to determine if the series is convergent or divergent. I used each test twice. Use the QR codes to determine the method I used. More than one method can work. You should get the same answer no matter the method.

7. $\sum_{n=1}^{\infty} \frac{1}{n2^n}$ converges by comparison b.c. $\frac{1}{2^n} > \frac{1}{n\cdot 2^n}$ & $\sum \left(\frac{1}{2}\right)^n$ converges by geometric $|R| = \frac{1}{2} < 1$

8. $\sum_{n=1}^{\infty} e^{-n}$

9. $\sum_{k=1}^{\infty} 2^{\frac{1}{k}}$

diverges by divergence b.c. $\lim_{n\to\infty} 2^{\frac{1}{n}} = 2^{\frac{1}{\infty}} = 2^0 = 1 \neq 0$

10. $\sum_{n=1}^{\infty} \frac{1}{n^{\frac{4}{3}}}$

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11. $\sum_{n=1}^{\infty} 10(0.25)^{n-1}$

converges by geometric $|R| = .25 < 1$.

12. $\sum_{n=5}^{\infty} \frac{1}{\sqrt{n-4}}$

13. $\sum_{n=1}^{\infty} \frac{n}{(n^2+1)^{\frac{3}{5}}}$

$u = n^2+1$

$du = 2n\,dn$

$\frac{1}{2}du = n\,dn$

$\lim_{R\to\infty} \frac{1}{2}\int_1^R u^{-3/5}du$

$\lim_{R\to\infty} \frac{1}{2}\frac{5}{2}u^{2/5}\Big|_1^R$

$\lim_{R\to\infty} \frac{5}{4}(n^2+1)^{2/5}\Big|_1^R$

$\lim_{R\to\infty} \frac{5}{4}(R^2+1)^{2/5} - \frac{5}{4}(2)^{2/5}$

$\infty - \#$

∞

diverges by Integral b.c. a_n = positive, decreasing & continuous & $\int a_n = \infty$

14. $\sum_{n=25}^{\infty} \frac{n}{n^2+1}$

15. $\sum_{n=1}^{\infty} n^{-\frac{1}{5}} = \sum_{n=1}^{\infty} \frac{1}{n^{1/5}}$

diverges by p-series $p = \frac{1}{5} \leq 1$.

16. $\sum_{n=1}^{\infty} \frac{n+1}{2n-1}$

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17.-20. Use the **integral test** to determine whether the series is convergent or divergent.

17. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$

diverges by the integral test b.c. a_n = pos, dec, cont & $\int a_n = \infty$

$\lim_{R\to\infty} \int_1^R n^{-1/5}\,dn$

$\lim_{R\to\infty} \frac{5n^{4/5}}{4}\Big|_1^R$

$\lim_{R\to\infty} \frac{5}{4}R^{4/5} - \frac{5}{4}$

$\infty - 5/4$

∞

18. $\sum_{n=1}^{\infty} \frac{1}{(2n+1)^3}$

19. $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

$u = n^2+1$
$du = 2n\,dn$
$\frac{1}{2}du = n\,dn$

diverges by Integral test b.c. a_n = pos, dec, & continuous and $\int a_n = \infty$

$\lim_{R\to\infty} \int_1^R \frac{1}{u}\,du$

$\lim_{R\to\infty} \ln|n^2+1|\Big]_1^R$

$\lim_{R\to\infty} \ln|R^2+1| - \ln 2$

$\infty - \ln 2$

∞

20. $\sum_{n=1}^{\infty} n^2 e^{-n^3}$

21.-26. Determine whether the series is convergent or divergent.

21. $\sum_{n=1}^{\infty} \frac{1}{n^{\sqrt{2}}}$ $\quad \sqrt{2} > 1$

converges by p-series
$p = \sqrt{2} > 1$.

22. $\sum_{n=1}^{\infty} n^{-0.9999}$

23. $1 + \frac{1}{8} + \frac{1}{27} + \frac{1}{64} + \frac{1}{125} + \ldots = \sum_{n=1}^{\infty} \frac{1}{n^3}$

$\frac{1}{1^3} \quad \frac{1}{2^3} \quad \frac{1}{3^3} \quad \frac{1}{4^3} \quad \frac{1}{5^3}$

converges by p-series
$p = 3 > 1$

24. $\sum_{n=1}^{\infty} \frac{1}{n^2+4}$

25. $\sum_{n=2}^{\infty}\frac{1}{n\ln(n)}$ $\quad u=\ln n$, $du=\frac{1}{n}dn$

$\lim_{R\to\infty}\int_2^R \frac{1}{u}\,du$

$\lim_{R\to\infty} \ln|\ln n|\Big]_2^R$

$\lim_{R\to\infty} \ln|\ln R|-\ln|\ln 2|$

$\infty-\ln|\ln 2|$

∞

diverges by Integral Test
a_n = pos, dec, & continuous
And $\int a_n=\infty$

26. $\sum_{n=1}^{\infty}\frac{e^{\frac{1}{n}}}{n^2}$

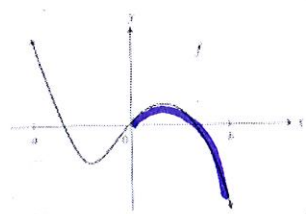
Review

R1. The graph of the function f is shown in the figure, which of the following statements are true?

I. $f'(0)=0$

II. f has an absolute maximum value on [a,b]

III. $f''<0$ on $(0,b)$



a. I b. II c. III d. I&II e. all

1. $f'(0)\neq 0$ Because f(x) does not have horizontal tangent (max/min)
2. No the max & min are Relative
3. Yes b.c. f(x) is concave down

R2. $\int\frac{1+x}{\sqrt{x}}dx = \int \frac{1}{x^{1/2}}+\frac{x}{x^{1/2}}\,dx$

a. $2\sqrt{x}+\frac{x^2}{2}+C$

b. $\frac{\sqrt{x}}{2}+\frac{3x^{\frac{3}{2}}}{2}+C$

c. $2\sqrt{x}+\frac{2x^{\frac{3}{2}}}{3}+C$

d. $x+\frac{2x^{\frac{3}{2}}}{3}+C$

e. 0

$\int x^{-1/2}+x^{1/2}\,dx$

$2x^{1/2}+\frac{2}{3}x^{3/2}+C$

$2\sqrt{x}+\frac{2}{3}x^{3/2}+C$

Answers

1. $\int=\frac{2}{9(25^3+9)^{\frac{3}{2}}}\therefore$ *converges*	2. $\int=\frac{1}{2e}\therefore$ *converges*	3. Diverges
4. Converges	5. Diverges	6. Converges
7. Converges	8. Converges	9. Diverges
10. Converges	11. Converges	12. Diverges
13. $\int=\infty\therefore$ *diverges*	14. $\int=\infty\therefore$ *diverges*	15. Diverges
16. Diverges	17. $\int=\infty\therefore$ *diverges*	18. $\int=\frac{1}{36}\therefore$ *converges*
19. $\int=\infty\therefore$ *diverges*	20. $\int=\frac{1}{3e}\therefore$ *converges*	21. Converges
22. Diverges	23. Converges	24. Converges
25. $\int=\infty\therefore$ *diverges*	26. $\int=e-1\therefore$ *converges*	R1. C R2. C