Homework Guide

BC Calculus
Geometric Series

Name Guide Pd.____
Infinite Series Day 2

Calculate the first eight terms of the sequence of the partial sums, correct to four decimal places. Does it appear that the series is convergent or divergent?

1. $\sum_{n=1}^{\infty} \frac{1}{n^3}$ Yes looks like converges to 1.2

$a_1 = \frac{1}{1^3} = 1$ $S_1 = 1$

$a_2 = \frac{1}{2^3} = \frac{1}{8} = .125$ $S_2 = 1.125$

$a_3 = \frac{1}{27} = .037$ $S_3 = 1.1620$

$a_4 = \frac{1}{64} = .0156$ $S_4 = 1.1777$

$a_5 = \frac{1}{125} = .008$ $S_5 = 1.1857$

$a_6 = \frac{1}{216} = .0046$ $S_6 = 1.1903$

$a_7 = \frac{1}{343} = .0029$ $S_7 = 1.1932$

$a_8 = \frac{1}{512} = .00195$ $S_8 = 1.19515$

2. $\sum_{n=1}^{\infty} \frac{n}{1+\sqrt{n}}$

Determine whether the geometric series is convergent or divergent. If it is convergent, find its sum.

3. $\sum_{n=1}^{\infty} 6(0.9)^{n-1}$ converges by geometric $|R| = .9 < 1.$

$\text{Sum} = \frac{6(.9)^0}{1-.9} = \frac{6}{.1} = 60$

4. $\sum_{n=1}^{\infty} \frac{10^n}{(-9)^{n-1}}$

5. $\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n}$ $\sum_{n=1}^{\infty} \frac{(-3)^n(-3)^{-1}}{4^n} = \sum_{n=1}^{\infty} -\frac{1}{3}\left(\frac{-3}{4}\right)^n$

Converges by geometric $|R| = 3/4 < 1.$

$S = \frac{\frac{(-3)^{1-1}}{4^1}}{\frac{4}{4}\cdot 1 + \frac{3}{4}} = \frac{\frac{1}{\cancel{4}}\frac{4}{7}}{\cancel{\frac{7}{4}}\frac{4}{7}} = \frac{1}{7}$

6. $\sum_{n=0}^{\infty} \frac{1}{(\sqrt{2})^n}$

Answers:

1. converges 2. diverges 3. 60 4. diverges 5. $\frac{1}{7}$ 6. $\frac{\sqrt{2}}{\sqrt{2}-1}$

Geometric Series

Infinite Series Day 2

7. $\sum_{n=0}^{\infty} \frac{\pi^n}{3^{n+1}} = \sum_{n=0}^{\infty} \frac{\pi^n}{3^n \cdot 3^1} = \sum_{n=0}^{\infty} \frac{1}{3}\left(\frac{\pi}{3}\right)^n$

diverges by geometric

$|R| = \frac{\pi}{3} \geq 1.$

8. $\sum_{n=1}^{\infty} \frac{e^n}{3^{n-1}}$

9. $\sum_{n=1}^{\infty} e^{-n} = \sum_{n=1}^{\infty} \frac{1}{e^n} = \sum_{n=1}^{\infty} \left(\frac{1}{e}\right)^n$

converges by geometric $|R| = \frac{1}{e} < 1$

$$\text{Sum} = \frac{\left(\frac{1}{e}\right)^1}{\frac{e}{e} \cdot 1 - \frac{1}{e}} = \frac{\frac{1}{e} \cdot \frac{e}{e-1}}{\frac{e-1}{e} \cdot \frac{e}{e-1}} = \frac{1}{e-1}$$

10. $\sum_{n=1}^{\infty} e^{3-2n}$

11. Find the value of c such that

$\sum_{n=0}^{\infty} e^{nc} = 10$

$$\sum_{n=0}^{\infty} (e^c)^n = 10$$

$$\text{Sum} = \frac{\text{first term}}{1 - \text{Ratio}}$$

$$10 = \frac{(e^c)^0}{1 - e^c}$$

$$10 = \frac{1}{1 - e^c}$$

Calculator

$\boxed{89}$ solve$(10 = 1 \div (1 - e \wedge (x)), x)$

$x = c = \ln\left(\frac{9}{10}\right)$

$\boxed{84}$ $Y_1 = \frac{1}{1 - e^x} - 10$ Find zero on calculator

$x = c = -.10536$

By Hand

$$\frac{10}{1} = \frac{1}{1 - e^c}$$

$$10(1 - e^c) = 1$$

$$\cancel{1} - e^c = \frac{1}{10}$$
$$-1 \qquad -1$$

$$-e^c = -\frac{9}{10}$$

$$e^c = \frac{9}{10}$$

$$\cancel{\ln e}^c = \ln\frac{9}{10}$$

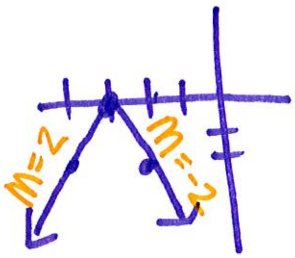
$$c = \ln\left(\frac{9}{10}\right)$$

Review

R1. If $g(x)=-2|x+3|$, what is the value $\lim_{x\to -3^-} g'(x)$?

A. -6 B. -2 C. 2

D. 6 E. nonexistent



x	$-2\lvert x+3\rvert$
-4	-2
-3	0
-2	-2

$\lim_{x\to -3} g'(x) = \text{dne}$

$\lim_{x\to -3^-} g'(x) = 2$ $\lim_{x\to -3^+} g'(x) = -2$

R2. What is $\lim_{\Delta x\to 0} \dfrac{\sin\left(\frac{\pi}{3}+\Delta x\right)-\sin\left(\frac{\pi}{3}\right)}{\Delta x}$

A. $-\frac{1}{2}$ B. 0 C. $\frac{1}{2}$

D. $\frac{\sqrt{3}}{2}$ E. Nonexistent

Reminder

$f'(\#) = \lim_{h\to 0} \dfrac{f(\#+h)-f(\#)}{h}$

Find $f'(\frac{\pi}{3})$ If $f(x)=\sin x$

$f'(x) = \cos x$

$f'(\pi/3) = \cos \pi/3 = \frac{1}{2}$

R3. If $f(x)$ is an antiderivative of xe^{-x^2} and $f(0)=1$, then $f(1)=$

A. $\frac{1}{e}$ B. $\frac{1}{2e}-\frac{3}{2}$ C. $\frac{1}{2e}-\frac{1}{2}$

D. $-\frac{1}{2e}+\frac{3}{2}$ E. $-\frac{1}{2e}+\frac{1}{2}$

$f(x) = \int xe^{-x^2}dx$

$u=-x^2$

$du=-2x\,dx$

$-\frac{1}{2}du = x\,dx$

$f(x) = -\frac{1}{2}\int e^u du$

$f(x) = -\frac{1}{2}e^{-x^2}+C$

$1 = -\frac{1}{2}e^0 + C$ $C=\frac{3}{2}$

$f(x) = -\frac{1}{2}e^{-x^2}+\frac{3}{2}$

$f(1) = -\frac{1}{2}e+\frac{3}{2}$

R4. If $g(x)=3\tan^2(2x)$, then $g'\left(\frac{\pi}{8}\right)$ is

A. 6 B. $6\sqrt{2}$ C. 12

D. $12\sqrt{2}$ E. 24

$g(x) = 3[\tan(2x)]^2$

$g'(x) = 6[\tan(2x)]^1 \cdot \sec^2(2x)\cdot 2$

$g'(x) = 12[\tan(2x)][\sec(2x)]^2$

$g'(\frac{\pi}{8}) = 12[\tan\frac{\pi}{4}][\sec\frac{\pi}{4}]^2$

$g'(\frac{\pi}{8}) = 12(1)(\frac{2}{\sqrt{2}})^2 = 24$

Answers:

7.	Diverges	8.	$\frac{3e}{3-e}$	9.	$\frac{1}{e-1}$	10.	$\frac{e^3}{e^2-1}$	11.	$c=\ln\left(\frac{9}{10}\right)$	R1	C	R2	C	R3	D	R4	E