BC Calculus
Review #3

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Infinite Series Day 11

III Sum = $\frac{\text{first term}}{1 - \text{Ratio}}$

1. Determine whether the series converges or diverges. If the series converges, find the sum.

a) $\sum_{n=0}^{\infty}\left(\frac{4}{5}\right)^n$ converges by geometric $|R| = \frac{4}{5} < 1$

$S = \frac{\left(\frac{4}{5}\right)^0}{1-\frac{4}{5}} = \frac{1}{\frac{1}{5}} = 5$ sum

b) $\sum_{n=3}^{\infty}\left(\frac{1}{3}\right)^n$ converges by geometric $|R| = 1/3 < 1$

$S = \frac{\left(\frac{1}{3}\right)^3}{1-\frac{1}{3}} = \frac{\frac{1}{27}}{\frac{2}{3}} = \frac{1}{18}$ sum

c) $\sum_{n=0}^{\infty}(.64)^n$ converges by geometric $|R| = .64 < 1$

$S = \frac{(.64)^0}{1-.64} = \frac{1}{.36}$

$S = \frac{100}{36} = \frac{25}{9}$

2. Determine whether each harmonic series converges or diverges. If it converges, find its sum.

a) $\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$

$\frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$

$1 = A(n+1) + B(n)$

$\boxed{\text{let } n=0}$ $1 = 1A$, $A = 1$ $\boxed{\text{let } n=-1}$ $1 = -B$, $B = -1$

$\sum_{n=1}^{\infty}\frac{1}{n} - \frac{1}{n+1}$

$\left(\frac{1}{1} - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) + \dots + \left(\frac{1}{\infty} - \frac{1}{\infty+1}\right)$

converges to 1

b) $\sum_{n=2}^{\infty}\frac{2}{n^2-1}$

$\frac{2}{(n+1)(n-1)} = \frac{A}{n+1} + \frac{B}{n-1}$

$2 = A(n-1) + B(n+1)$

$\boxed{\text{let } n=-1}$ $2 = -2A$, $A = -1$ $\boxed{\text{let } n=1}$ $2 = 2B$, $B = 1$

$\sum_{n=2}^{\infty}\frac{-1}{n+1} + \frac{1}{n-1}$

n=2, n=3, n=4, n=5

$\left(-\cancel{\frac{1}{3}} + 1\right) + \left(-\cancel{\frac{1}{4}} + \frac{1}{2}\right) + \left(-\cancel{\frac{1}{5}} + \cancel{\frac{1}{3}}\right) + \left(-\cancel{\frac{1}{6}} + \cancel{\frac{1}{4}}\right) + \dots$

$+ \left(\frac{-1}{\infty+1} + \frac{1}{\infty-1}\right)$ → 0, 0

converges to 3/2

3. Determine whether the series converges or diverges.

a) $\sum_{n=1}^{\infty}\frac{1}{n}$

diverges by p-series
$p = 1 \le 1$.

b) $\sum_{n=1}^{\infty}\frac{1}{n^2}$

converges by p-series
$p = 2 > 1$.

c) $\sum_{n=1}^{\infty}\ln\left(\frac{2n+1}{n-3}\right)$

diverges by divergence

$\lim_{n\to\infty} \ln\left(\frac{2n+1}{n-3}\right) = \ln(2) \neq 0$

d) $\sum_{n=2}^{\infty}\frac{1}{n\ln n}$

$u = \ln(n)$ $du = \frac{1}{n}du$

$\lim_{R\to\infty}\int_2^R \frac{1}{u}du$

$\lim_{n\to\infty} \ln|\ln n|\Big]_2^R$ $\lim_{n\to\infty} \ln|\ln R| - \ln|\ln 2|$

diverges by Integral $a_n = \frac{1}{n \cdot \ln n}$ is positive, decreasing, & cont. & $\int a_n = \infty$

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e) $\sum_{n=3}^{\infty} \frac{1}{n-3}$ $\frac{1}{n} < \frac{1}{n-3}$

& $\sum_{n=3}^{\infty} \frac{1}{n}$ diverges by p-series $p=1\le 1$.

diverges by comparison

f) $\sum_{n=1}^{\infty} \frac{1}{3^n+1}$ $\frac{1}{3^n} > \frac{1}{3^n+1}$

& $\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n$ converges by geometric $|R| = \frac{1}{3} < 1$

Converges by comparison

g) $\sum_{n=1}^{\infty} \frac{1}{n^2+n+1}$ $b_n = \frac{1}{n^2}$

$\lim_{n\to\infty} \frac{1}{n^2+n+1} \cdot \frac{n^2}{1} = 1 > 0$

& $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by p-series $p=2>1$.

Converges by limit comparison

h) $\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$ $b_n = \frac{n^2}{n^3} = \frac{1}{n}$

$\lim_{n\to\infty} \frac{n^2}{n^3+1} \cdot \frac{n}{1} = 1 > 0$

& $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges by p-series $p=1\le 1$.

diverges by limit comparison

i) $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^3-1}}$ $\frac{1}{n^{3/2}} = b_n$

$\lim_{n\to\infty} \frac{1}{\sqrt{n^3-1}} \cdot \frac{\sqrt{n^3}}{1} = 1 > 0$

& $\sum_{n=2}^{\infty} \frac{1}{n^{3/2}}$ converges by p-series $p=3/2>1$

converges by limit comparison

j) $\sum_{n=1}^{\infty} \frac{n^2-1}{n^4+1}$ $\frac{1}{n^2} = b_n$

$\lim_{n\to\infty} \frac{n^2-1}{n^4+1} \cdot \frac{n^2}{1} = 1 > 0$

& $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by p-series $p=2>1$.

Converges by limit comparison

k) $\sum_{n=2}^{\infty} \frac{n^2-n+1}{(n+1)(n-1)}$

diverges by divergence

$\lim_{n\to\infty} \frac{n^2-n+1}{n^2-1} = 1 \neq 0$

l) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$

$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$ is alternating

$\lim_{n\to\infty} \frac{1}{n} = 0$

converges by alternating series

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m) $\sum_{n=1}^{\infty}(-1)^n \cdot \frac{2n+3}{4n-1}$

let n = even let n = odd

$\lim_{n\to\infty} \frac{2n+3}{4n-1} = \frac{1}{2}$ $\lim_{n\to\infty} -\frac{2n+3}{4n-1} = -\frac{1}{2}$

diverges by divergence test

$\lim_{n\to\infty} \frac{(-1)^n 2n+3}{4n-1} = \text{dne} \neq 0.$

n) $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{\sqrt{n}}$ is alternating

$\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$

converges by Alternating Series test

o) $\sum_{n=1}^{\infty}\left|\frac{n^3}{2^n}\right|$ $R = \lim_{n\to\infty} \frac{(n+1)^2}{2^{n+1}} \cdot \frac{2^n}{n^2}$

$R = \lim_{n\to\infty} \frac{\cancel{2^n}(n+1)^2}{\cancel{2^n}\cdot 2^1 n^2} = \frac{1}{2} < 1$

converges by Ratio $|R| = \frac{1}{2} < 1$

p) $\sum_{n=0}^{\infty}\left|\frac{2^n}{n!}\right|$ $R = \lim_{n\to\infty} \frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n}$

$R = \lim_{n\to\infty} \frac{\cancel{2^n}\cdot 2^1 \cdot \cancel{n!}}{\cancel{2^n}\cdot(n+1)\cdot \cancel{n!}} = 0 < 1$

Converges by Ratio $|R| = 0 < 1$.

q) $\sum_{n=0}^{\infty}\frac{(n+1)!}{n!}$ ~~$R = \lim_{n\to\infty} \frac{(n+2)!}{(n+1)!} \cdot \frac{n!}{(n+1)!}$~~

~~$R = \lim_{n\to\infty} \frac{(n+2)(n+1)!\cdot n!}{(n+1)!\cdot(n+1)\cdot n!} = 1$~~ $\boxed{\text{Inconclusive}}$

$\sum_{n=0}^{\infty} \frac{(n+1)\cancel{n!}}{\cancel{n!}}$ $\lim_{n\to\infty} n+1 = \infty$

diverges by divergence $\lim_{n\to\infty} = \infty \neq 0$

r) $\sum_{n=1}^{\infty}\left|\frac{n^4}{3^n}\right|$ $R = \lim_{n\to\infty} \frac{(n+1)^4}{3^{n+1}} \cdot \frac{3^n}{n^4}$

$R = \lim_{n\to\infty} \frac{\cancel{3^n}(n+1)^4}{\cancel{3^n}\cdot 3^1 \cdot n^4} = \frac{1}{3} < 1$

converges by Ratio $|R| = \frac{1}{3} < 1$

Determine if each sequence converges absolutely, conditionally, or not at all.

a) $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}$ $\boxed{\text{Positive}}$ $\sum_{n=1}^{\infty}\frac{1}{n}$ diverges by p-series $p = 1 \leq 1$.

$\boxed{\text{Alternating}}$ $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$ is alternating

& $\lim_{n\to\infty}\frac{1}{n} = 0$ Converges by alternating

Converges conditionally

b) $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2}$ $\boxed{\text{Positive}}$ $\sum_{n=1}^{\infty}\frac{1}{n^2}$

converges by p-series $p = 2 > 1$.

Converges absolutely

c) $\sum_{n=0}^{\infty}(-1)^n\frac{n}{n^2+1}$ $\boxed{\text{Positive}}$ $\sum_{n=1}^{\infty}\frac{n}{n^2+1}$ $b_n = \frac{1}{n}$ $\lim_{n\to\infty}\frac{n}{n^2+1}\cdot\frac{n}{1} = 1 > 0$

$\sum_{n=1}^{\infty}\frac{1}{n}$ diverges by p-series $p = 1 \leq 1$

diverges by limit comp.

$\boxed{\text{Alternating}}$ $\sum_{n=0}^{\infty}\frac{(-1)^n n}{n^2+1}$ is alternating & $\lim_{n\to\infty}\frac{n}{n^2+1} = 0$ $\therefore$ converges by Alternating

Converges Conditionally

$f(x)=\frac{2}{9}\left(\sqrt{x^3+1}\right)^3-6$

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$f(0)=\frac{2}{9}-6=\frac{2}{9}-\frac{54}{9}=-\frac{52}{9}$

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R1. If f(x) is an antiderivative of $x^2\sqrt{x^3+1}$ and f(2)=0, then f(0)=

a. -6
b. 6
c. $\frac{2}{9}$
d. $-\frac{52}{9}$
e. $\frac{56}{9}$

$u=x^3+1$
$du=3x^2dx$
$\frac{1}{3}du=x^2dx$

$f(x)=\int x^2\sqrt{x^3+1}\,dx$

$f(x)=\frac{1}{3}\int u^{1/2}du$

$f(x)=\frac{1}{3}\cdot\frac{2}{3}u^{3/2}+C$

$f(x)=\frac{2}{9}\left(\sqrt{x^3+1}\right)^3+C$

$0=\frac{2}{9}(27)+C$

$C=-6$

R2. If $f(x)=5\cos^2(\pi-x)$, then $f'\left(\frac{\pi}{2}\right)$ is

a. 0
b. -5
c. 5
d. -10
e. 10

$f(x)=5[\cos(\pi-x)]^2$

$f'(x)=10[\cos(\pi-x)](-\sin(\pi-x))(-1)$

$10\cos(\pi-x)\sin(\pi-x)$

$10\cos\frac{\pi}{2}\sin\frac{\pi}{2}$

0

R3. $g(x)=\int_1^x \frac{3t}{t^3+1}dt$, then g'(2) is

a. 0
b. $-\frac{2}{3}$
c. $\frac{2}{3}$
d. $-\frac{5}{6}$
e. $\frac{5}{6}$

$g'(x)=\cancel{\frac{d}{dx}\int_1^x}\frac{3t}{t^3+1}$

$g'(x)=\frac{3x}{x^3+1}$

$g'(2)=\frac{6}{9}=\frac{2}{3}$

R4. If $\int_k^2(2x-2)dx=-3$ a possible value for k is

a. -2
b. 0
c. 1
d. 2
e. 3

$x^2-2x\Big|_k^2=-3$

$(\cancel{4-4})-(k^2-2k)=-3$

$k^2-2k-3=0$

$(k+1)(k-3)$

$k=-1 \quad k=3$

Answers

1. a. 5 b. $\frac{1}{18}$ c. $\frac{100}{36}$
2. a. Convers to 1 b. Converges to $\frac{3}{2}$
3. a. D b. C c. D d. D e. D f. C g. C h. D i. C j. C k. D l. C m. D n. C o. C p. C q. D r. C
4. a. Conditionally b. Absolutely c. Absolutely

R1. D R2. A R3. C R4. ~~A~~ E