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Review #2 Infinite Series Day 10

Determine if the series is convergent or divergent. Indicate which Series Test you used to find your answer.

1. $\sum_{n=0}^{\infty}\left(\frac{2}{3}\right)^n$

converges by geometric $|R| = \frac{2}{3} < 1$.

2. $\sum_{n=0}^{\infty}\frac{2}{n^2+1}$ $\frac{2}{n^2} > \frac{2}{n^2+1}$

$\sum_{n=1}^{\infty}\frac{2}{n^2}$ converges by p-series $p = 2 > 1$.

Converges by comparison

3. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$ Alternating

& $\lim_{n\to\infty}\frac{1}{n} = 0$

Converges by alternating

4. $\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n}} = \sum_{n=1}^{\infty}\frac{1}{n^{1/3}}$

diverges by p-series $p = 1/3 < 1$.

5. $\sum_{n=0}^{\infty}\frac{2n+1}{3n+2}$

diverges by divergence

$\lim_{n\to\infty}\frac{2n+1}{3n+2} = \frac{2}{3} \neq 0$.

6. $\sum_{n=4}^{\infty}\frac{\ln n}{n^4}$ $\frac{n}{n^4} > \frac{\ln n}{n^4}$

& $\sum_{n=4}^{\infty}\frac{n}{n^4}$ converges by p-series $p = 3 > 1$.

Converges by comparison

7. $\sum_{n=0}^{\infty}\frac{1}{\sqrt{n^3+1}}$ $\frac{1}{n^{3/2}} > \frac{1}{\sqrt{n^3+1}}$

& $\sum_{n=1}^{\infty}\frac{1}{n^{3/2}}$ converges by p-series $p = 3/2 > 1$

converges by comparison

8. $\sum_{n=1}^{\infty}\frac{(-1)^n(n+1)}{n-5}$

let n = even $\lim_{n\to\infty}\frac{n+1}{n-5} = 1$

let n = odd $\lim_{n\to\infty} -\frac{(n+1)}{n-5} = -1$

diverges by divergence

$\lim_{n\to\infty}\frac{(-1)^n(n+1)}{n-5} = \text{dne} \neq 0$

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9. $\sum_{n=0}^{\infty} \frac{2^n}{n^3}$ $R = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$

$R = \lim_{n\to\infty} \frac{2^{n+1}}{(n+1)^3} \cdot \frac{n^3}{2^n} = \lim_{n\to\infty} \frac{\cancel{2^n} \cdot 2^1 n^3}{\cancel{2^n}(n+1)^3} = 2$

diverges by Ratio
$|R| = 2 > 1.$

10. $\sum_{n=1}^{\infty} \frac{\ln n}{n}$ $u = \ln n$
$du = \frac{1}{n} dn$

$\lim_{R\to\infty} \int_1^R u\,du \quad \lim_{R\to\infty} \frac{1}{2}(\ln n)^2 \Big|_1^R$

$\lim_{R\to\infty} \frac{1}{2}(\ln(R))^2 - \frac{1}{2}(\ln(1))^2 = \infty$

diverges by Integral $a_n = \frac{\ln(n)}{n}$ is pos, dec, & continuous $[1,\infty)$ & $\int a_n = \infty$

11. $\sum_{n=0}^{\infty} \frac{2^n}{n!}$ $R = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$

$\lim_{n\to\infty} \frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n} = \lim_{n\to\infty} \frac{\cancel{2^n} \cdot 2^1 \cdot \cancel{n!}}{(n+1)\cancel{n!} \cdot \cancel{2^n}} = 0$

Converges by Ratio $|R| = 0 < 1$

12. $\sum_{n=0}^{\infty} \frac{n!}{e^n}$ $R = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$

$\lim_{n\to\infty} \frac{(n+1)!}{e^{n+1}} \cdot \frac{e^n}{n!} = \lim_{n\to\infty} \frac{(n+1) \cdot \cancel{n!} \cdot \cancel{e^n}}{\cancel{e^n} \cdot e^1 \cdot \cancel{n!}} = \infty$

diverges by Ratio $|R| = \infty > 1$

13. $\sum_{n=0}^{\infty} e^{-n} = \sum_{n=0}^{\infty} \frac{1}{e^n} = \sum_{n=0}^{\infty} \left(\frac{1}{e}\right)^n$

converges by geometric
$|R| = \frac{1}{e} < 1$

14. $\sum_{n=1}^{\infty} \left(\frac{2n}{5n-1}\right)^n$

$L = \lim_{n\to\infty} \sqrt[n]{\left(\frac{2n}{5n-1}\right)^n} = \lim_{n\to\infty} \frac{2n}{5n-1} = \frac{2}{5}$

converges by Root
$L = \frac{2}{5} < 1$

15. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

converges by alternating
$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ is alternating
& $\lim_{n\to\infty} \frac{1}{n^2} = 0$

16. $\sum_{n=1}^{\infty} \frac{n}{\sqrt[3]{n^7 + n^4}}$ $b_n = \frac{n}{n^{7/3}} = \frac{1}{n^{4/3}}$

$\lim_{n\to\infty} \frac{n}{\sqrt[3]{n^7+n^4}} \cdot \frac{n^{7/3}}{n} = 1 > 0$

& $\sum_{n=1}^{\infty} \frac{1}{n^{4/3}}$ converges by p-series $p = 4/3 > 1.$

converges by limit comparison

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17. $\sum_{n=1}^{\infty} \frac{(n^2+3n-4)}{n!}$

$R = \lim_{n\to\infty} \frac{(n+1)^2+3(n+1)-4}{(n+1)!} \cdot \frac{n!}{n^2+3n+4}$

$= \lim_{n\to\infty} \frac{[(n+1)^2+3(n+1)-4]\cancel{n!}}{(n+1)\cdot\cancel{n!}(n^2+3n-4)} \frac{n^2}{n^3} = 0$

converges by Ratio $|R| = 0 < 1$

18. $\sum_{n=1}^{\infty} \frac{1}{2+3^n}$

$\frac{1}{3^n} > \frac{1}{3^n+2}$

& $\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n$ converges by geometric $|R| = \frac{1}{3} < 1$

19. $\sum_{n=3}^{\infty} \frac{n}{(4+n^2)^{3/4}}$

$b_n = \frac{n}{(n^{\cancel{2}})^{3/\cancel{4}2}} = \frac{n}{n^{3/2}} = \frac{1}{n^{1/2}}$

$\lim_{n\to\infty} \frac{n}{(4+n^2)^{3/4}} \cdot \frac{(n^2)^{3/4}}{n} = 1 > 0$

& $\sum_{n=3}^{\infty} \frac{1}{n^{1/2}}$ diverges by p-series $p = 1/2 < 1$

diverges by limit comparison

20. $\sum_{n=2}^{\infty} \frac{(2n)!}{(n-1)3^n}$

$R = \lim_{n\to\infty} \frac{(2n+2)!}{n\cdot 3^{n+1}} \cdot \frac{(n-1)3^n}{(2n)!}$

$\lim_{n\to\infty} \frac{\cancel{3^n}(2n+2)(2n+1)\cancel{(2n)!}(n-1)}{\cancel{3^n}\cdot 3^1\cancel{(2n)!}\,n} = \infty$

diverges by Ratio $|R| = \infty > 1$

21. Determine whether each series converges absolutely, converges conditionally, or diverges. Show your work.

a. $\sum_{n=1}^{\infty} \frac{(-1)^n (n-1)^3}{(n+1)^3}$

Positive $\sum_{n=1}^{\infty} \frac{(n-1)^3}{(n+1)^3} = 1 \neq 0$

diverges by divergence

Alternating

let n=even $\lim_{n\to\infty} \frac{(n-1)^3}{(n+1)^3} = 1$

let n=odd $\lim_{n\to\infty} -\frac{(n-1)^3}{(n+1)^3} = -1$

diverges by divergence

$\lim_{n\to\infty} \frac{(-1)^n(n-1)^3}{(n+1)^3} = \text{dne} \neq 0$

diverges

b. $\sum_{n=1}^{\infty} \frac{(-1)^n}{2^{n+2}}$

$\sum_{n=1}^{\infty} \frac{(-1)^n}{(2)^n\cdot(2)^2} = \sum_{n=1}^{\infty} \left(\frac{-1}{2}\right)^n \cdot \frac{1}{4}$

Converges by geometric $|R| = \frac{1}{2} < 1$

Converges absolutely

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R1. $\int \frac{x^4-1}{x^2}dx = \int \frac{x^4}{x^2} - \frac{1}{x^2}$

$\int x^2 - x^{-2}$

$\frac{x^3}{3} - \frac{x^{-1}}{-1} + C$

$\frac{x^3}{3} + \frac{1}{x} + C$

a. $\frac{x^3}{3}+x+C$

b. $\frac{x^3}{3}-x+C$

c. $\frac{x^3}{3}+\frac{3}{x^3}+C$

d. $\frac{x^3}{3}+\frac{1}{x}+C$

e. $\frac{x^3}{3}-\frac{1}{x}+C$

$P(x) = \int q(x)$

R2. If $\int p'(x) = \int q(x)$ and q is continuous function for all values of x, then $\int_{-1}^{0} q(4x)dx$ is

a. $p(0)-p(-4)$

b. $4p(0)-4p(-4)$

c. $\frac{1}{4}p(0)-\frac{1}{4}p(-4)$

d. $\frac{1}{4}p(0)+\frac{1}{4}p(-4)$

e. $p(0)+p(-4)$

$\frac{p(4x)}{4}\Big|_{-1}^{0}$

$\frac{p(0)}{4} - \frac{p(4(-1))}{4}$

R3. Water is leaking from a tank at a rate represented by f(t) whose graph is shown in the figure to the right. Which of the following is the best approximation of the total amount of water leaked from the tank for $1 \le t \le 3$?

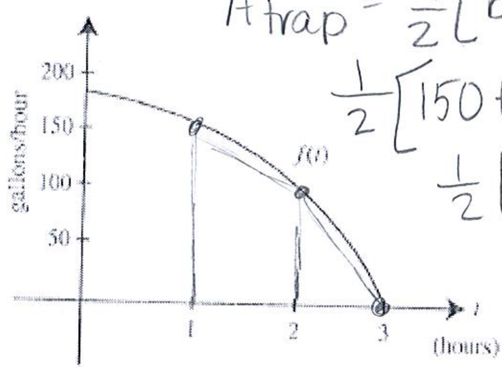
a. $\frac{9}{2}$ *gallons*

b. 5 *gallons*

c. 175 *gallons*

d. 350 *gallons*

e. 450 *gallons*



$A_{trap} = \frac{h}{2}[b_1 + b_2]$

$\frac{1}{2}[150 + 100 + 100 + 0]$

$\frac{1}{2}[350]$

175

Answers:

1. C	2. C	3. C	4. D	5. D
6. C	7. C	8. D	9. D	10. D
11. C	12. D	13. C	14. C	15. C
16. C	17. C	18. C	19. D	20. D
21a. D	21.b Absolute	R1. D	R2. C	R3. C