Name **Review – Applications of Derivatives** $\frac{76}{80}$ (with 3 and 5 written beside)

Linearization Non-Calculator On Test

Find a linear approximation for each quantity.

1. Approximate $\sqrt{8.98} \approx 2\frac{299}{300}$

$f(x)=\sqrt{x} \quad a=9$

Tangent line

$\boxed{1}$ Point $f(9)=\sqrt{9}=3$

$\boxed{2}$ Slope $f'(9)=\frac{1}{2\sqrt{9}}=\frac{1}{2\cdot 3}=\frac{1}{6}$

$f(x)=x^{1/2}$

$f'(x)=\frac{1}{2}x^{-1/2}=\frac{1}{2\sqrt{x}}$

$y-3=\frac{1}{6}(x-9)$

$y=\frac{1}{6}(x-9)+3$

$y(8.98)=\frac{1}{6}(8.98-9)+3$

$y(8.98)=\frac{-.02}{6}+3=-\frac{2}{600}+3=3-\frac{1}{300}$

2. Approximate $\sqrt[5]{31} \approx 1\frac{79}{80}$

$f(x)=\sqrt[5]{x} \quad a=32$

Tangent Line

$\boxed{1}$ Point $f(32)=\sqrt[5]{32}=2$

$\boxed{2}$ Slope $f'(32)=\frac{1}{5(\sqrt[5]{32})^4}=\frac{1}{5(2)^4}=\frac{1}{80}$

$f(x)=x^{1/5}$

$f'(x)=\frac{1}{5}x^{-4/5}=\frac{1}{5x^{4/5}}=\frac{1}{5(\sqrt[5]{x})^4}$

$y-2=\frac{1}{80}(x-32)$

$y=\frac{1}{80}(x-32)+2$

$y(31)=\frac{1}{80}(31-32)+2$

$y(31)=-\frac{1}{80}+2=1\frac{79}{80}$

3. Approximate $\ln(0.9) \approx -.1$

$f(x)=\ln x \quad a=1$

Tangent line

$\boxed{1}$ Point $f(1)=\ln(1)=0$

$\boxed{2}$ Slope $f'(1)=\frac{1}{1}=1$

$f(x)=\ln x$

$f'(x)=\frac{1}{x}$

$y-0=1(x-1)$

$y=1(x-1)$

$y(.9)=1(.9-1)$

$y(.9)=1(-.1)$

$y(.9)=-.1$

4. Approximate $e^{-0.1} \approx .9$

$f(x)=e^x \quad a=0$

Tangent line

$\boxed{1.}$ Point $f(0)=e^0=1$

$\boxed{2.}$ Slope $f'(0)=e^0=1$

$f(x)=e^x$

$f'(x)=e^x$

$y-1=1(x-0)$

$y=1(x-0)+1$

$y(-.1)=1(-.1-0)+1$

$y(-.1)=-.1+1$

$y(-.1)=.9$

Absolute Extrema Non- Calculator on Test

5. Find all absolute extrema in the interval [0, 2π] for $y = x + \cos x$.

$y = x + \cos x$
$y' = 1 - \sin x$
$0 = 1 - \sin x$
$\sin x = 1$
$x = \frac{\pi}{2}$

x	y
0	1
$\frac{\pi}{2}$	0
2π	$2\pi+1$

$y(0) = 0 + \cos(0) = 1$
$y(\frac{\pi}{2}) = 0 + \cos\frac{\pi}{2} = 0$
$y(2\pi) = 2\pi + \cos(2\pi) = 2\pi+1$

Abs min of 0 at $\pi/2$ Abs max of $2\pi+1$ at 2π

6. Find the absolute maximum and absolute minimum of $f(x) = x^2 - 4x + 7$ on the interval [0, 3].

$f(x) = x^2 - 4x + 7$
$f'(x) = 2x - 4$
$0 = 2x - 4$
$2x = 4$
$x = 2$

x	y
0	7
2	3
3	4

$f(0) = 0^2 - 4(0) + 7 = 7$
$f(2) = 4 - 8 + 7 = 3$
$f(3) = 9 - 12 + 7 = 4$

Abs max of 7 at 0 Abs min of 3 at 2

7. Find the minimum and maximum values of $f(x) = x^2 - 2x + 1$ on the interval [0, 3].

$f(x) = x^2 - 2x + 1$
$f'(x) = 2x - 2$
$0 = 2x - 2$
$2x = 2$
$x = 1$

x	y
0	1
1	0
3	4

$f(0) = 1$
$f(1) = 1 - 2 + 1 = 0$
$f(3) = 9 - 6 + 1 = 4$

Abs min of 0 at 1 Abs max of 4 at 3

8. Find the absolute extrema of the function $f(x) = \frac{x}{x^2 - x + 1}$ on the interval [0, 3].

$f'(x) = \frac{(x^2-x+1)(1) - x(2x-1)}{(x^2-x+1)^2}$

$f'(x) = \frac{x^2 - \cancel{x} + 1 - 2x^2 + \cancel{x}}{(x^2-x+1)^2}$

$f'(x) = \frac{-x^2+1}{(x^2-x+1)^2}$

$f'(x) = 0$
$-x^2 + 1 = 0$
$x^2 = 1$
$x = \pm 1$

$f'(x) = \text{ud}$
$\cancel{(x^2-x+1)^2} = 0$
garbage

x	y
0	0
1	1
3	$\frac{3}{7}$

$f(0) = \frac{0}{1} = 0$
$f(1) = \frac{1}{1-1+1} = 1$
$f(3) = \frac{3}{9-3+1} = \frac{3}{7}$

Abs min of 0 at 0
Abs max of 1 at 1

Calculator on all the Rest!

Optimization

9. Express the number 10 as a sum of two nonnegative numbers whose product is as large as possible.

$\boxed{x = 5 \quad y = 5}$

2 non-negative numbers

$x + y = 10$

$y = 10 - x$

$y = 10 - 5$

$y = 5$

Product = maximum

Product = xy

Product = $x(10 - x)$

Product = $10x - x^2$

Product' = $10 - 2x$

$0 = 10 - 2x$

$2x = 10$

$x = 5$

$\nearrow + \quad 5 \quad - \searrow$

$P'(4) = +$

$P'(6) = -$

10. A rectangular field is to be bounded by a fence on three sides and by a straight stream on the fourth side. Find the dimensions of the field with maximum area that can be enclosed with 1000 feet of fence.

Stream

w w

l

$\boxed{l = 500 \quad w = 250}$

Perimeter = 1000

$2w + l = 1000$

$l = 1000 - 2w$

$l =$

Area = maximum

Area = $l \cdot w$

Area = $(1000 - 2w)w$

Area = $1000w - 2w^2$

Area' = $1000 - 4w$

$0 = 1000 - 4w$

$4w = 1000$

$w = 250$

$\nearrow + \quad 250 \quad - \searrow$

$A'(249) = +$

$A'(251) = -$

11. A rectangular plot of land is to be fenced in using two kinds of fencing. Two opposite sides will use heavy-duty fencing selling for $3 a foot, while the remaining two sides will use standard fencing selling for $2 a foot. What are the dimensions of the rectangular plot of greatest area that can be fenced in at a cost of $6000?

l

w

$\boxed{l = 500 \quad w = 750}$

Cost = 6000

Cost = $3 \cdot 2 \cdot l + 2 \cdot 2 \cdot w$

$6000 = 6l + 4w$

$6l = 6000 - 4w$

$l = 1000 - \frac{2}{3}w$

$l = 500$

Area = maximum

Area = $l \cdot w$

Area = $(1000 - \frac{2}{3}w)w$

Area = $1000w - \frac{2}{3}w^2$

Area' = $1000 - \frac{4}{3}w$

$0 = 1000 - \frac{4}{3}w$

$\frac{4}{3}w = 1000$

$4w = 3000$

$w = 750$

$\nearrow + \quad 750 \quad - \searrow$

$A'(749) = +$

$A'(751) = -$

12. A rectangular area of 3200 ft^2 is to be fenced off. Two opposite sides will use fencing costing $1 per foot and the remaining sides will use fencing costing $2 per foot. Find the dimensions of the rectangle of least cost.

l

w

$\boxed{l = 40 \quad w = 80}$

Area = 3200

$l \cdot w = 3200$

$l = \frac{3200}{w}$

$l = \frac{3200}{80} = 40$

Cost = minimum

Cost = $1(2)(w) + 2(2)l$

Cost = $2w + 4l$

Cost = $2w + 4\left(\frac{3200}{w}\right)$

Cost = $2w + 12800w^{-1}$

Cost' = $2 - 12800w^{-2}$

$\frac{12800}{w^2} = 2$

$2w^2 = 12800$

$w^2 = 6400$

$w = 80$

$\searrow - \quad 80 \quad + \nearrow$

$C'(79)$ $C'(81)$

$x = y^2$

13. Find the point on the parabola $y = \sqrt{x}$ that is closest to the point (3, 0).

Points

1 (3, 0)

2 $(x, \sqrt{x})$

(y^2, y)

$\left(\frac{5}{2}, \sqrt{\frac{5}{2}}\right)$

distance = minimum

distance = $\sqrt{(x-3)^2 + (\sqrt{x} - 0)^2}$

distance = $\sqrt{x^2 - 6x + 9 + x}$

distance = $(x^2 - 5x + 9)^{1/2}$

distance' = $\frac{1}{2}(x^2 - 5x + 9)^{-1/2}(2x - 5)$

distance' = $\frac{2x - 5}{2\sqrt{x^2 - 5x + 9}}$

$0 = 2x - 5$

$x = 5/2$

$\searrow - \quad 5/2 \quad + \nearrow$

$d'(0)$ $d'(3)$

14. A gardener wishes to create two equal sized gardens by enclosing a rectangular area with 300 feet of fencing and fence it down the middle. What is the largest rectangular area that may be enclosed? $l \cdot w$

$\boxed{\text{Area} = \underline{3750}}$

$l = 75 \quad w = 50$

Perimeter = 300

$2l + 3w = 300$

$2l = 300 - 3w$

$l = 150 - \frac{3}{2}w$

$l = 150 - \frac{3}{2}(50) = 75$

Area = maximum

Area = $l \cdot w$

Area = $(150 - \frac{3}{2}w)w$

Area = $150w - \frac{3}{2}w^2$

Area′ = $150 - 3w$

$0 = 150 - 3w$

$3w = 150$

$w = 50$

$\nearrow 50 \searrow$ (+ | −)

$A'(49) \quad A'(51)$

→ 15. Find the point on the graph of $y = \sqrt{x+4}$ that is at a minimum distance to the point (4, -2). $y = \sqrt{x+4}$

$y^2 = x + 4$

$x = y^2 - 4$

$\boxed{1}$ (4, -2)

$\boxed{2}$ $(x, \cancel{\sqrt{x+4}})$

$(y^2 - 4, y)$

$\boxed{(3.124, 2.669)}$

distance = $\sqrt{(y^2 - 4 - 4)^2 + (y+2)^2}$

distance = $\sqrt{(y^2-8)^2 + (y+2)^2}$

distance = $\sqrt{y^4 - 16y^2 + 64 + y^2 + 4y + 4}$

distance = $(y^4 - 15y^2 + 4y + 68)^{1/2}$

distance′ = $\frac{1}{2}(y^4 - 15y^2 + 4y + 68)^{-1/2}(4y^3 - 30y + 4)$

distance′ = $\frac{4y^3 - 30y + 4}{2\sqrt{y^4 - 15y^2 - 4y + 68}}$

distance = minimum

$4y^3 - 30y + 4 = 0$

$y = \cancel{-2.802}, .134, 2.669$

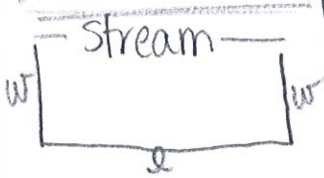
$- \searrow + \nearrow - \searrow + \nearrow$

Find zero using calc

y-can't be neg

16. A rectangular field is to be bounded by a fence on three sides and by a straight stream on the fourth side. Find the dimensions of the field with maximum area that can be enclosed using 1000 ft of fence.

$\boxed{l = \underline{500} \quad w = \underline{250}}$



Perimeter = 1000

$2w + l = 1000$

$l = 1000 - 2w$

$l = 1000 - 2(250)$

$l = 500$

Area = maximum

$A = l \cdot w$

$A = (1000 - 2w)w$

$A = 1000w - 2w^2$

$A' = 1000 - 4w$

$4w = 1000$

$w = 250$

$\nearrow 250 \searrow$ (+ | −)

$A'(249) \quad A'(251)$

17. Find the point(s) on the graph of the curve $y = x^2 + 1$ that is closest to the point (4, 1).

$\boxed{1}$ (4, 1)

$\boxed{2}$ $(x, x^2 + 1)$

$\boxed{(1.128, 2.273)}$

distance = $\sqrt{(x-4)^2 + (x^2 \cancel{+1} \cancel{-1})^2}$

distance = $\sqrt{x^2 - 8x + 16 + x^4}$

distance′ = $\frac{1}{2}(x^4 + x^2 - 8x + 16)^{-\frac{1}{2}}(4x^3 + 2x - 8)$

distance′ = $\frac{4x^3 + 2x - 8}{2\sqrt{x^4 + x^2 - 8x + 16}}$

$4x^2 + 2x - 8 = 0$

$x = 1.1281739$

$- \searrow , \nearrow +$

$d'(0) \quad d'(2)$

distance = minimum

Find zero using Calc.

18. A closed rectangular container with a square base is to have a volume of 2250 in³. The material for the top and bottom of the container will cost \$2 per in², and the material for the sides will cost \$3 per in². Find the dimensions of the container of least cost.

$\boxed{l = \underline{15} \quad w = \underline{15} \quad h = \underline{10}}$

Volume = 2250

$x^2 y = 2250$

$y = \frac{2250}{x^2}$

Cost = minimum

Cost = \$2(2)(Area top) + \$3(4)(Side area)

Cost = $4x^2 + 12xy$

Cost = $4x^2 + 12x\left(\frac{2250}{x^2}\right)$

Cost = $4x^2 + 27000x^{-1}$

$C' = 8x - 27000x^{-2}$

$\frac{27000}{x^2} = 8x$

$27000 = 8x^3$

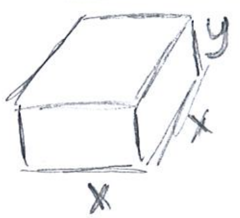
$x = 15$

$- \searrow 15 \nearrow +$

$C'(14) \quad C'(16)$

19. A container with square base, vertical sides, and open top is to be made from 1000 ft² of material. Find the dimensions of the container with greatest volume.

$l = 18.257$ $w = 18.257$ $h = 9.129$



$SA = 1000$

$x^2 + 4xy = 1000$

$4xy = 1000 - x^2$

$y = \frac{250}{x} - \frac{x}{4}$

III Volume = max

$V = x^2 y$

$V = x^2\left(\frac{250}{x} - \frac{x}{4}\right)$

$V = 250x - \frac{1}{4}x^3$

$V' = 250 - \frac{3}{4}x^2$

$\frac{4}{3} \cdot \frac{3}{4}x^2 = 250 \cdot \frac{4}{3}$

$x^2 = \frac{1000}{3}$

$x = 18.257418$

18.257: + | −

$V'(17)$ $V'(19)$

20. Find the points on the parabola $y = 4 - x^2$ that are at a minimum distance to the point (0, -3).

1 $(0,-3)$

2 $(x, 4-x^2)$

$\boxed{\left(\pm\sqrt{\frac{13}{2}}, -\frac{5}{2}\right)}$

$4 - \left(\pm\sqrt{\frac{13}{2}}\right)^2$

$\frac{8}{2} - \frac{13}{2}$

III minimum = distance

$\text{distance} = \sqrt{(x-0)^2 + (4-x^2+3)}$

$\text{distance} = \sqrt{x^2 + (7-x^2)^2}$

$\text{distance} = \sqrt{x^2 + 49 - 14x^2 + x^4}$

$\text{distance} = (x^4 - 13x^2 + 49)^{1/2}$

$d' = \frac{1}{2}(x^4 - 13x^2 + 49)^{-1/2}(4x^3 - 26x)$

$0 = \frac{4x^3 - 26x}{2\sqrt{x^4 - 13x^2 + 49}}$

$2x(2x^2 - 13) = 0$

$2x = 0$ $2x^2 - 13 = 0$

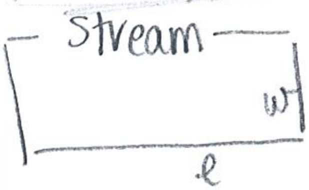
$x = 0$ $x = \pm\sqrt{13/2}$

$-\sqrt{13/2}$ 0 $\sqrt{13/2}$: − | + | − | +

$d'(-4)$ $d'(-1)$ $d'(1)$ $d'(4)$

21. A rectangular field is to be bounded by a fence on three sides and by a straight stream on the fourth side. Find the dimensions of the field with maximum area that can be enclosed with 1000 feet of fence.

$l = 500$ $w = 250$



Perimeter = 1000

$2w + l = 1000$

$l = 1000 - 2w$

$l = 1000 - 2(250)$

$l = 500$

III Area = maximum

$A = l \cdot w$

$A = (1000 - 2w)w$

$A = 1000w - 2w^2$

$A' = 1000 - 4w$

$4w = 1000$

$w = 250$

250: + | −

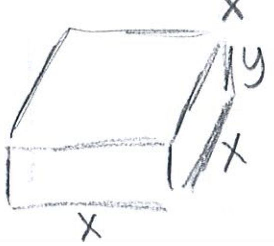
$A'(249)$ $A'(251)$

22. A closed metal box has a square base and top. The square base and top cost \$2 per square meter, but the other faces cost \$4 per square meter. Find the minimum cost of such a box having a volume of 4 m³.

Cost = \$48

$l = 2$ $w =$ ___ $h =$ ___

$\text{Cost} = 4x^2 + \frac{64}{x}$



$V = 4$

$x^2 y = 4$

$y = \frac{4}{x^2}$

Cost = minimum

$\text{Cost} = \$2(2)x^2 + \$4(4)xy$

$\text{Cost} = 4x^2 + 16xy$

$\text{Cost} = 4x^2 + 16x\left(\frac{4}{x^2}\right)$

$\text{Cost} = 4x^2 + 64x^{-1}$

$\text{Cost}' = 8x - 64x^{-2}$

$\frac{64}{x^2} = 8x$

$8x^3 = 64$

$x^3 = 8$

$x = 2$

Related Rates

23. A ladder 25 feet long is leaning against the wall of a house. The base of the ladder is pulled away from the wall at a rate of 2 feet per second. How fast is the top of the ladder moving down the wall when its base is 7 feet?

Know: $\frac{dx}{dt} = 2\frac{ft}{sec}$

Find: $\frac{dy}{dt} =$

When: $x = 7ft$

Eqn: $\left[x^2 + y^2 = 25^2\right]\frac{d}{dt}$

$7^2 + y^2 = 25^2$

$y = 24$

Der: $2x\frac{dy}{dt} + 2y\frac{dy}{dt} = 0$

Sub: $2(7ft)\left(2\frac{ft}{sec}\right) + 2(24)\frac{dy}{dt} = 0$

$28\frac{ft^2}{sec} + 48ft\frac{dy}{dt} = 0$

$\frac{1}{\cancel{48ft}}\cancel{48ft}\frac{dy}{dt} = -28\frac{ft^2}{sec}\frac{1}{48ft}$

$\boxed{\frac{dy}{dt} = \frac{-7}{12}\frac{ft}{sec}}$

24. A spherical balloon is inflated so that its volume is increasing at a rate of 3 ft^3/min. How fast is the radius of the balloon increasing at the instant the radius is $\frac{1}{2}$ ft?

Know: $\frac{dV}{dt} = \frac{3ft^3}{min}$

Find: $\frac{dR}{dt} =$

When: $R = \frac{1}{2}ft$

Eqn: $\left[V = \frac{4}{3}\pi R^3\right]\frac{d}{dt}$

Der: $\frac{dV}{dt} = \frac{4}{3}\pi(3R^2)\frac{dR}{dt}$

$\frac{dV}{dt} = 4\pi R^2\frac{dR}{dt}$

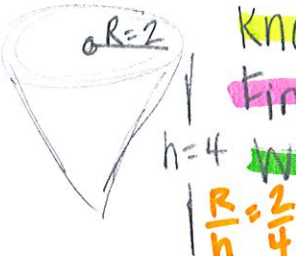
$\frac{3ft^3}{min} = 4\pi\left(\frac{1}{2}ft\right)^2\frac{dR}{dt}$

$\frac{3ft^3}{min} = \cancel{4}\pi\left(\frac{1}{\cancel{4}}\right)ft^2\frac{dR}{dt}$

$\frac{1}{\pi\,ft^2}\frac{3ft^{\cancel{3}}}{min} = \frac{1\cancel{\pi\,ft^2}}{\cancel{\pi\,ft^2}}\frac{dR}{dt}$

$\boxed{\frac{dR}{dt} = \frac{3}{\pi}\frac{ft}{min}}$

24. The diameter and height of a paper cup in the shape of a cone are both 4 inches, and water is leaking out at the rate of 0.5 in^3/sec. Find the rate at which the water level is dropping when the diameter of the surface is 2 inches.



Know: $\frac{dV}{dt} = \frac{-1}{2}\frac{in^3}{sec}$

Find: $\frac{dh}{dt} =$ ____

When: $d = 2$

$R = 1$

$h = 2$

Eqn: $V = \frac{1}{3}\pi R^2 h$

Get Rid of R

$4R = 2h \quad R = \frac{1}{2}h$

$V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h$

$V = \frac{1}{3}\pi\,\frac{1}{4}h^2 \cdot h$

$\frac{d}{dt}\left[V = \frac{1}{12}\pi h^3\right]$

Der:

$\frac{dV}{dt} = \frac{1}{12}\pi(3h^2)\frac{dh}{dt}$

$\frac{dV}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt}$

Sub:

$-\frac{1}{2}\frac{in^3}{sec} = \frac{\pi}{4}(2in)^2\frac{dh}{dt}$

$-\frac{1}{2}\frac{in^3}{sec} = \frac{\pi}{\cancel{4}}(\cancel{4}in^2)\frac{dh}{dt}$

$\frac{1}{\pi\cancel{in^2}}\frac{-1}{2}\frac{in^{\cancel{3}}}{sec} = \cancel{\pi\,in^2}\frac{dh}{dt}\frac{1}{\cancel{\pi\,in^2}}$

$\boxed{\frac{dh}{dt} = \frac{-1}{2\pi}\frac{in}{sec}}$

26. The radius r of a circle is increasing at a rate of 3 centimeters per minute. Find the rate of change of the area of the circle when the radius is 24 centimeters.

Know: $\frac{dR}{dt} = 3\frac{cm}{min}$

Find: $\frac{dA}{dt} =$

When: $R = 24cm$

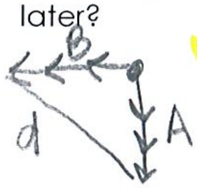
Eqn: $\left[A = \pi R^2\right]\frac{d}{dt}$

Der: $\frac{dA}{dt} = \pi(2R)\frac{dR}{dt}$

Sub: $\frac{dA}{dt} = 2\pi(24cm)\left(\frac{3cm}{min}\right)$

$\boxed{\frac{dA}{dt} = 144\frac{\pi cm^2}{min}}$

27. Two cars start moving from the same point. One travels south at 60 miles per hour and the other travels west at 25 miles per hour. At what rate is the distance between the cars increasing two hours later?



Know: $\frac{dA}{dt} = 60\frac{mi}{hR}$

$\frac{dB}{dt} = 25\frac{mi}{hR}$

Find: $\frac{dd}{dt} =$

When: $t = 2$

$A = 60\frac{mi}{\cancel{hR}}(2\cancel{hR}) = 120mi$

$B = 25\frac{mi}{hR}(2hR) = 50mi$

$120^2 + 50^2 = d^2$

$d = 130$

Eqn: $\left[A^2 + B^2 = d^2\right]\frac{d}{dt}$

$\cancel{2}A\frac{dA}{dt} + \cancel{2}B\frac{dB}{dt} = \cancel{2}d\frac{dd}{dt}$

$\frac{dd}{dt} = \frac{A\frac{dA}{dt} + B\frac{dB}{dt}}{d}$

Sub: $\frac{dd}{dt} = \frac{120(60) + 50(25)}{130}$

$\boxed{\frac{dd}{dt} = 65\frac{mi}{hR}}$

28. A water tank has the shape of an inverted circular cone with base radius of 2 feet and a height of 4 feet. If water is being pumped into the tank at a rate of 2 ft^3/min, find the rate at which the water level is rising when the water is 3 meters deep.

$\frac{r}{h} = \frac{2}{4}$

Know $\frac{dV}{dt} = 2 \frac{ft^3}{min}$

Find $\frac{dh}{dt} =$

When $h = 3ft$

Eqn: $V = \frac{1}{3}\pi R^2 h$

Get Rid of R

$4R = 2h \quad R = \frac{1}{2}h$

$V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h$

$\left[V = \frac{1}{12}\pi \cdot h^3\right]\frac{d}{dt}$

Der: $\frac{dV}{dt} = \frac{1}{12}\pi(3h^2)\frac{dh}{dt}$

Sub: $\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$\frac{2\ ft^3}{min} = \frac{1}{4}\pi(3ft)^2\frac{dh}{dt}$

$\frac{2ft^3}{min} = \frac{\pi 9ft^2}{4}\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{2ft^3}{min} \cdot \frac{4}{9\pi ft^2}$

$\boxed{\frac{dh}{dt} = \frac{8}{9\pi}\frac{ft}{min}}$

29. Water is falling on a surface, wetting a circular area that is expanding at a rate of 8 mm^2/s. How fast is the radius of the wetted area expanding when the radius is 171 mm?

Know: $\frac{dA}{dt} = 8\frac{mm^2}{sec}$

Find: $\frac{dR}{dt} =$

When: $R = 171mm$

Eqn: $\left[A = \pi R^2\right]\frac{d}{dt}$

Der: $\frac{dA}{dt} = \pi(2R)\frac{dR}{dt}$

Sub: $\frac{8mm^2}{sec} = 2\pi(171mm)\frac{dR}{dt}$

$\frac{1}{342\pi mm}\ \frac{8mm^2}{sec} = \cancel{342\pi mm}\frac{dR}{dt}\ \frac{1}{\cancel{342\pi mm}}$

$\boxed{\frac{dR}{dt} = \frac{4}{171\pi}\frac{mm}{sec}}$

30. If a snowball (perfect sphere) melts so that its surface area decreases at a rate of 1 centimeter squared per minute, find the rate at which the diameter decreases when the diameter is 10 centimeters.

Know: $\frac{dSA}{dt} = -\frac{1cm^2}{min}$

Find: $\frac{dd}{dt}$

When: $d = 10cm$

Eqn: $SA = 4\pi R^2$

Get Rid of R

$R = \frac{1}{2}d$

$SA = 4\pi\left(\frac{1}{2}d\right)^2$

$SA = \cancel{4}\pi\ \frac{1}{\cancel{4}} \cdot d^2$

$\left[SA = \pi d^2\right]\frac{d}{dt}$

Der: $\frac{dSA}{dt} = \pi(2d)\frac{dd}{dt}$

$\frac{dSA}{dt} = 2\pi d\frac{dd}{dt}$

Sub $\frac{1}{20\pi cm}\ -\frac{1cm^2}{min} = \cancel{2\pi(10cm)}\frac{dd}{dt}\ \frac{1}{\cancel{20\pi cm}}$

$\boxed{\frac{dd}{dt} = -\frac{1}{20\pi}\frac{cm}{min}}$

31. The area of a circular region is increasing at a rate of 96π square meters per second. When the area of the region is 64π m, how fast, in m/sec, is the radius of the region increasing?

Know: $\frac{dA}{dt} = 96\pi\frac{m^2}{sec}$

Find: $\frac{dR}{dt} =$

When: $A = 64\pi$

$A = \pi R^2$

$64\cancel{\pi} = \cancel{\pi}R^2 \quad R = 8$

Eqn: $\left[A = \pi R^2\right]\frac{d}{dt}$

Der: $\frac{dA}{dt} = \pi(2R)\frac{dR}{dt}$

$\frac{dA}{dt} = 2\pi R\frac{dR}{dt}$

Sub: $\frac{1}{16\cancel{\pi}\cancel{m}}\ 96\cancel{\pi}\frac{m^{\cancel{2}}}{sec} = \cancel{2\pi(8m)}\frac{dR}{dt}\ \frac{1}{\cancel{16\pi m}}$

$\boxed{\frac{dR}{dt} = \frac{6m}{sec}}$

32. Boats N and W are headed to the same port. Boat N is 5 miles south of the port, heading due north at 10 miles per hour. Boat W is 11 miles east of the port, heading due west at 7 miles per hour. At what rate is the distance between the boats changing?

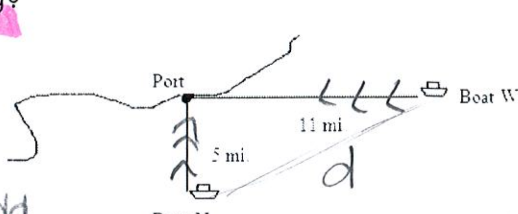
Know: $\frac{dN}{dt} = -\frac{10mi}{hR}$ $\quad \frac{dW}{dt} = -\frac{7mi}{hR}$

Find: $\frac{dd}{dt} =$

When: $N = 5mi$

$W = 11mi$

$d = \sqrt{11^2 + 5^2}$



Eqn: $\left[N^2 + W^2 = d^2\right]\frac{d}{dt}$

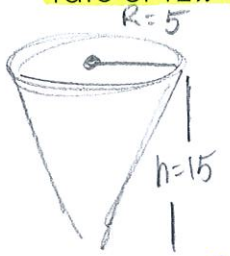
Der: $\cancel{2}N\frac{dN}{dt} + \cancel{2}W\frac{dW}{dt} = \cancel{2}d\frac{dd}{dt}$

$\frac{dd}{dt} = \frac{N\frac{dN}{dt} + W\frac{dW}{dt}}{d}$

Sub: $\frac{dd}{dt} = \frac{5(-10) + 11(-7)}{\sqrt{146}}$

$\boxed{\frac{dd}{dt} = -10.511\frac{mi}{hR}}$

33. A cone has radius 5 cm and height 15 cm. It is empty and is being filled with water at a constant rate of 12π cm³/s. Find the rate of change of the radius when the radius of the water is 2 cm.



Know: $\frac{dV}{dt} = 12\pi \frac{cm^3}{sec}$

Find: $\frac{dR}{dt} =$

When: $R = 2cm$

Eqn: $V = \frac{1}{3}\pi R^2 h$

Get Rid of h

$\frac{R}{h} = \frac{5}{15}$ $\quad 5h = 15R \quad h = 3R$

$V = \frac{1}{\cancel{3}}\pi R^2 (\cancel{3}R)$

$\left[V = \pi R^3\right] \frac{d}{dt}$

Der: $\frac{dV}{dt} = \pi (3R^2)\frac{dR}{dt}$

$\frac{dV}{dt} = 3\pi R^2 \frac{dR}{dt}$

Sub: $12\pi \frac{cm^3}{sec} = 3\pi (2cm)^2 \frac{dR}{dt}$

$\frac{1}{12\pi cm^2}\ \frac{\cancel{12\pi} cm^{\cancel{3}}}{sec} = \cancel{12\pi cm^2}\frac{dR}{dt}\ \frac{1}{12\pi cm^2}$

$\boxed{\frac{dR}{dt} = 1 \frac{cm}{sec}}$

34. Cars A and B leave a town at the same time. Car A heads due south at a rate of 80 km/hr and Car B heads due west at a rate of 60 km/hr. How fast is the distance between the cars increasing after three hours?

Know: $\frac{dA}{dt} = 80 \frac{km}{hr}$

$\frac{dB}{dt} = 60 \frac{km}{hr}$

Find: $\frac{dd}{dt}$

When: $t = 3$

$A = 80(3) = 240km$

$B = 60(3) = 180km$

$d = \sqrt{240^2 + 180^2}$

$d = 300$

Eqn: $\left[A^2 + B^2 = d^2\right] \frac{d}{dt}$

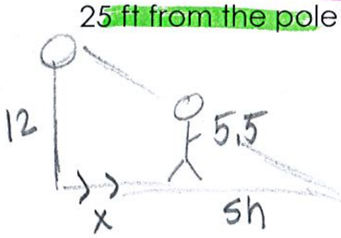
Der: $\cancel{2}A\frac{dA}{dt} + \cancel{2}B\frac{dB}{dt} = \cancel{2}d\frac{dd}{dt}$

$\frac{dd}{dt} = \frac{A\frac{dA}{dt} + B\frac{dB}{dt}}{d}$

Sub: $\frac{dd}{dt} = \frac{(240)(80) + (180)(60)}{300}$

$\boxed{\frac{dd}{dt} = 100 \frac{km}{hr}}$

35. A light is on the top of a 12 ft tall pole and a 5ft 6in tall person is walking away from the pole at a rate of 2 ft/sec. At what rate is the tip of the shadow moving away from the pole when the person is 25 ft from the pole?



5ft 6in $= 5\frac{6}{12} = 5.5ft$

Know: $\frac{dx}{dt} = 2\frac{ft}{sec}$

Find: $\frac{dsh}{dt} + \frac{dx}{dt} =$ tip shadow

$\frac{dsh}{dt} =$

When: $x = 25ft$

Eqn: $\frac{12}{5.5} = \frac{x + sh}{sh}$

$12sh = 5.5x + 5.5sh$

$\left[6.5sh = 5.5x\right] \frac{d}{dt}$

Der: $6.5\frac{ds}{dt} = 5.5\frac{dx}{dt}$

Sub: $6.5\frac{ds}{dt} = 5.5(2)$

$\frac{ds}{dt} = \frac{22}{13}$

$\boxed{\text{tip of shadow} = \frac{22}{13} + 2 = \frac{48}{13}}$

36. Two boats leave the same port at the same time with one boat traveling north at 15 knots per hour and the other boat traveling west at 12 knots per hour. How fast is the distance between the two boats changing after 2 hours?

Know: $\frac{dN}{dt} = 15 \frac{knots}{hr}$

$\frac{dW}{dt} = 12 \frac{knots}{hr}$

Find: $\frac{dd}{dt} =$ ___

When: $t = 2hrs$

$N = 15(2) = 30$

$W = 12(2) = 24$

$d = \sqrt{30^2 + 24^2}$

$d = \sqrt{1476}$

Eqn $\left[N^2 + W^2 = d^2\right] \frac{d}{dt}$

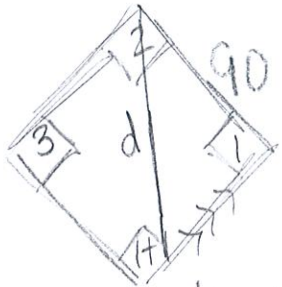
Der: $\cancel{2}N\frac{dN}{dt} + \cancel{2}W\frac{dW}{dt} = \cancel{2}d\frac{dd}{dt}$

$\frac{dd}{dt} = \frac{N\frac{dN}{dt} + W\frac{dW}{dt}}{d}$

Sub: $\frac{dd}{dt} = \frac{30(15) + 24(12)}{\sqrt{1476}}$

$\boxed{\frac{dd}{dt} = 19.209 \frac{knots}{hr}}$

37. The baseball diamond from Field of Dreams is a square with side 90 ft. A ghost batter hits the ball and runs toward first base with a speed of 24 ft/s. At what rate is his distance from second base changing when he is halfway to first base?



~~When: x=45ft~~

Eqn: $\left[x^2+90^2=d^2\right]\frac{d}{dt}$

Der: $\cancel{2}x\frac{dx}{dt}=\cancel{2}d\frac{dd}{dt}$

Know: $\frac{dx}{dt}=24\frac{ft}{sec}$

Find: $\frac{dd}{dt}$

$\frac{dd}{dt}=\frac{x}{d}\frac{dx}{dt}$

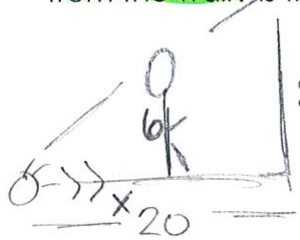
Sub: $\frac{dd}{dt}=\frac{45\,\cancel{ft}}{\sqrt{10125}\,\cancel{ft}}\left(-\frac{24ft}{sec}\right)$

$45^2+90^2=d^2$

$d=\sqrt{10125}$

$\boxed{\frac{dd}{dt}=-10.733}$

38. A spot light is on the ground 20 ft away from a wall and a 6 ft tall person is walking towards the wall at a rate of 2.5 ft/sec. How fast is the height of the shadow changing when the person is 8 feet from the wall? Is the shadow increasing or decreasing in height at this time?



~~When: 8ft~~ from wall

$x=12$

Eqn: $\frac{x}{6}=\frac{20}{sh}$

$\left[x\cdot sh=120\right]\frac{d}{dt}$

Der: $x\frac{dsh}{dt}+sh\frac{dx}{dt}=0$

Know: $\frac{dx}{dt}=2.5\frac{ft}{sec}$

Find: $\frac{dsh}{dt}$

$\frac{dsh}{dt}=-\frac{sh}{x}\frac{dx}{dt}$

$\frac{12}{6}=\frac{20}{sh}$

$12sh=120$

$sh=10$

Sub: $\frac{dsh}{dt}=-\frac{10}{12}(2.5)$

$\boxed{\frac{dsh}{dt}=-\frac{25}{12}\frac{ft}{sec}}$

39. A baseball diamond has the shape of a square, and each side is 90 feet long. A player is running from second to third base and he is 60 feet from reaching third. He is running at a speed of 25 feet per second. At what rate is the player's distance from home plate decreasing?

Eqn: $\left[x^2+90^2=d^2\right]\frac{d}{dt}$

$60^2+90^2=d^2$

$d=\sqrt{11700}$

Der: $\cancel{2}x\frac{dx}{dt}=\cancel{2}d\frac{dd}{dt}$

$\frac{dd}{dt}=\frac{x}{d}\frac{dx}{dt}$

Know: $\frac{dx}{dt}=-25\frac{ft}{sec}$

Sub: $\frac{dd}{dt}=\frac{60}{\sqrt{11700}}\left(-\frac{25ft}{sec}\right)$

Find: $\frac{dd}{dt}=$ ___

When: $x=60$ ft

$\boxed{\frac{dd}{dt}=-13.867}$