Review: Polar/Parametric/Vector Name______________________________

1. Area of Polar $\frac{1}{2}\int_{\alpha}^{\beta} R^2\, d\theta$	2. Area Between 2 Curves Polar $\frac{1}{2}\int_{\alpha}^{\beta} (R_2)^2-(R_1)^2\, d\theta$	3. How do you find horizontal tangent lines? top of $\frac{dy}{dx}=0$ and solve	Polar 4. $\frac{dy}{dx}=\frac{\frac{dR}{d\theta}\sin\theta+R\cos\theta}{\frac{dR}{d\theta}\cos\theta-R\sin\theta}$
5. $\sin 2\theta =$ $2\sin\theta\cos\theta$	~~Parametric~~ 6. Arc Length over [a, b] $\int_a^b \sqrt{1+f'(x)^2}\,dx$	make sure you have all three 7. $\cos 2\theta =$ $\cos^2 x-\sin^2 x$ $2\cos^2 x-1$ $1-2\sin^2 x$	8. How do you find vertical tangent lines? bottom of $\frac{dy}{dx}=0$ and solve
Parametric 9. $\frac{d^2y}{dx^2}=$ $\frac{x'(t)y''(t)-y'(t)x''(t)}{[x'(t)]^3}$	Parametric 10. Arc Length over [a, b] $\int_a^b \sqrt{[x'(t)]^2+[y'(t)]^2}$	11. How do you convert from polar to rectangular points? $x=R\cos\theta$ $y=R\sin\theta$	Rectangular $\rightarrow$ Parametric 12. a.) Line through point (a,b) with slope m $c(t)=(a+t, b+mt)$ b.) Circle center(a,b) $c(\theta)=(a+R\cos\theta, b+R\sin\theta)$

No calculators may be used on this portion of the test.

Suppose the curve defined by the parameterization $c(t)=\left(3t+3, t^4-8t^2\right)$. Answer the following.

1. Find the slope of the curve for $t=1$.

$$\frac{dy}{dx}=\frac{4t^3-16t^2}{3}=\frac{4(1)^3-16(1)^2}{3}=\frac{4-16}{3}=\frac{-12}{3}=-4$$

2. ~~If possible, find the points, in Cartesian form, corresponding to each horizontal and vertical tangent.~~ Tangent Line for $t=1$

$\boxed{\text{Point}}$

$$c(1)=(3(1)+3, (1)^4-8(1)^2)$$
$$=(6,-7)$$

$$y--7=-4(x-6)$$
$$y+7=-4(x-6)$$

* You will have to find H/V tangents

Name ______________________

Answer the following.

$\frac{dR}{d\theta} = 2(-\sin\theta) = -2\sin\theta$

5. Find an equation for the line tangent to the curve $r = 3 + 2\cos\theta$ at $\theta = \frac{\pi}{2}$. Write your answer in Cartesian form.

$\frac{dy}{dx} = \frac{-2\sin\theta\sin\theta + (3+2\cos\theta)\cos\theta}{-2\sin\theta\cos\theta - (3+2\cos\theta)\sin\theta} = \frac{-2(1)(1) + \cancel{(3+2(0))(0)}^{\to 0}}{-2(1)(0) - (3+2(0))(1)} = \frac{-2}{-3} = \frac{2}{3}$

$pt(0,3)$

$m = 2/3$

$y - 3 = \frac{2}{3}(x - 0)$

$X = R\cos\theta \quad y = R\sin\theta$

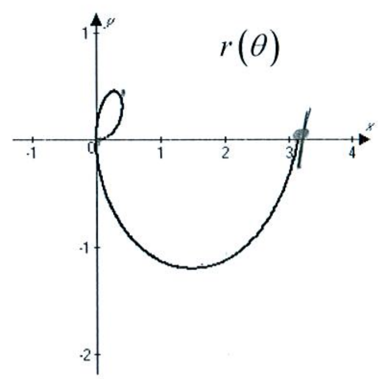
$X = (3+2\cos\theta)\cos\theta \quad y = (3+2\cos\theta)\sin\theta$

$X = (3+2(0))(0) \quad y = (3+2(0))(1)$

$X = 0 \quad y = 3$

Suppose the graph of $r(\theta) = \theta\cos\theta$ for $0 \le \theta \le \pi$ given at the right. Answer the following.

$\frac{dR}{d\theta} = \theta(-\sin\theta) + \cos\theta(1)$



6. Calculate $\frac{dr}{d\theta}$. Then, evaluate your result for $\theta = \pi$.

$\frac{dR}{d\theta} = \cos\theta - \theta\sin\theta$

$= \cos\pi - \pi\sin\pi$

$= -1 - \pi(0)$

$= -1$

7. Interpret your result for $\left.\frac{dr}{d\theta}\right|_{\theta=\pi}$ from #6 with respect to the curve. Explain your reasoning.

θ	$\theta\cos\theta$
0	0
$\pi/4$	$\pi/4 \cdot \sqrt{2}/2$
$\pi/2$	0
$\frac{3\pi}{4}$	$\frac{3\pi}{4}(-\sqrt{2}/2)$
π	$-\pi$
$5\pi/4$	

Skip :)

Review: Polar/Parametric/Vector Name____________________

Calculators may be used on this portion of the test.

Suppose a particle's velocity along a path is described by the vector $\vec{v}(t)=\langle 2t-2,\ t^2-t^4\rangle$. Answer the following. Show all steps in your work.

8. Calculate the speed of the particle at $t = 0.1$.

$\text{Speed} = \sqrt{[v(x)]^2+[v(y)]^2} = \sqrt{(2t-2)^2+(t^2-t^4)^2} = 1.80003$

9. Set up and evaluate an expression to find the total distance traveled by the particle on the interval $0 \le t \le 1$.

$\int_0^1 \sqrt{(2t-2)^2+(t^2-t^4)^2} = \frac{428}{315} \approx 1.0231$

10. Is the particle speeding up or slowing down at time $t = 0.1$? Justify your answer.

$v(.1) = \langle -1.8, .0099\rangle$

$a(t) = \langle 2, 2t-4t^3\rangle = \langle 2, .196\rangle$

dot product $= (-1.8)(2)+(.0099)(.196)$

-3.59806

slowing down

Suppose a particle's position along a path is described by the vector $\vec{r}(t)=\langle 1-\sin t,\ \cos t\rangle$ for $0 \le t < 2\pi$. Answer the following. Show all steps in your work.

11. Find $\vec{v}(t)$ and $\vec{a}(t)$.

$\vec{v}(t) = \langle -\cos t, -\sin t\rangle$

$\vec{a}(t) = \langle \sin t, -\cos t\rangle$

12. Is there a time for $0 \le t < 2\pi$ when the particle is not moving? If so, when? If not, explain your reasoning.

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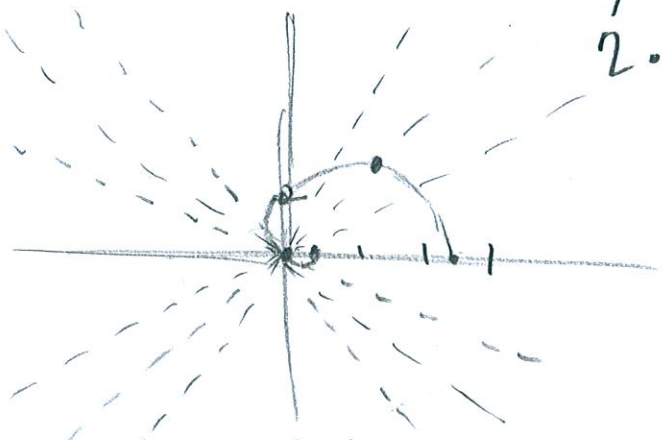
Answer the following. Be sure to include your set up.

13. Find the area enclosed by the inner loop of the curve defined by $r = 1 + \sqrt{2}\cos\theta$.

θ	
0	2.4
$\pi/6$	2.2
$\pi/4$	2
$\pi/3$	1.7
$\pi/2$	1
$2\pi/3$	$.29$
$3\pi/4$	0
$5\pi/6$	$-.2$
π	$-.4$

bottom half of loop

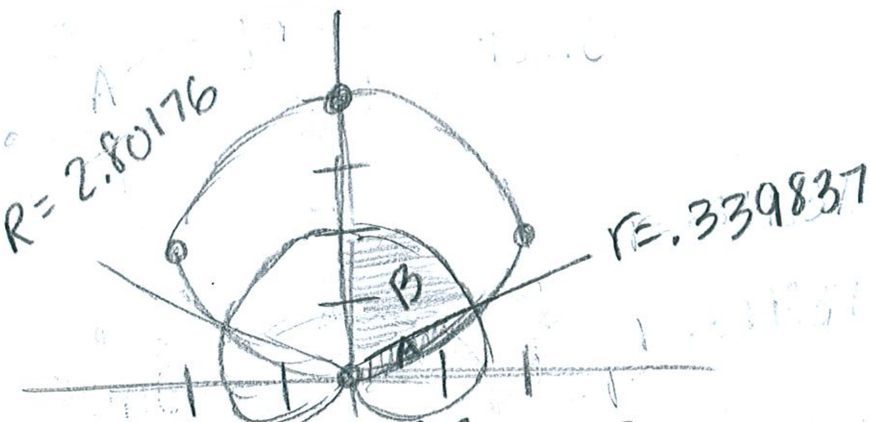


to get top half

$$2 \cdot \frac{1}{2}\int_{\frac{3\pi}{4}}^{\pi} (1+\sqrt{2}\cos\theta)^2\, d\theta$$

$$\frac{\pi}{2} - \frac{3}{2} \approx .070796$$

14. Find the area of the region that lies inside both of the curves defined by $r = 4\sin\theta$ and $r = 1 + \sin\theta$.



$$A = \frac{1}{2}\int_0^{.339873} (4\sin\theta)^2\, d\theta \qquad B = \frac{1}{2}\int_{.339837}^{\pi/2} (1+\sin\theta)^2\, d\theta$$

$.102269$ $\qquad$ 1.9446

$$2(2.04680) = \boxed{4.09373}$$

$1 + \cancel{\sin\theta} = 4\sin\theta$

$\cancel{-\sin\theta} \quad -\sin\theta$

$3\sin\theta = 1$

$\theta = \sin^{-1}\left(\frac{1}{3}\right)$

$.339837$

OR

$$\frac{1}{2}\int_0^{.339837} (4\sin\theta)\, d\theta + \frac{1}{2}\int_{.339837}^{2.80176} (1+\sin\theta)^2\, d\theta + \frac{1}{2}\int_{2.80176}^{\pi} (4\sin\theta)^2\, d\theta$$

$$.102269 + 3.8892 + .10226$$

$$\boxed{4.09373}$$