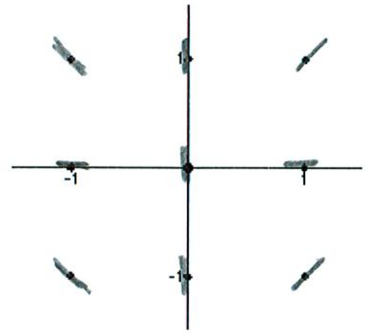
AP Calculus- BC
Review for Additional Differentiation Test

Name________________________

No calculators

Use the differential equation $\frac{dy}{dx} = \frac{y^2}{x^3}$

1. On the graph shown at the right, sketch a slope field for the differential equation at the indicated points.



$(1,1) = 1/1 = 1$ $\quad (0,1) = \frac{1}{0} = ud$ $\quad (-1,1) = \frac{(1)^2}{(-1)^3} = -1$

$(1,0) = \frac{0}{1} = 0$ $\quad (0,0) = ud$ $\quad (-1,0) = \frac{0}{-1} = 0$

$(1,-1) = (-1)^2/1 = 1$ $\quad (0,-1) = \frac{-1}{0} = ud$ $\quad (-1,-1) = \frac{1}{-1} =$

2. Solve the differential equation given the initial condition $y(1) = 2$

$x^3 dy = y^2 dx$

$\frac{1}{y^2} dy = \frac{1}{x^3} dx$

$y^{-2} dy = x^{-3} dx$

$\frac{y^{-1}}{-1} = \frac{x^{-2}}{-2} + C$

$\frac{-1}{y} = \frac{-1}{2x^2} + C$

$\frac{-1}{2} = \frac{-1}{2(1)^2} + C$

$C = 0$

$\frac{-1}{y} = \frac{-1}{2x^2}$

$\frac{1}{y} = \frac{1}{2x^2}$

$y = 2x^2$

3. Use Euler's Method to approximate $y(1.5)$ given $y(1) = 2$ and $\Delta x = 0.1$. Be sure to include your calculations in a table.

$y(1) = \quad = 2$

$y(1.1) = 2 + .1[2^2/1^3] = 2.4$

$y(1.2) = 2.4 + .1[(2.4)^2/(1.1)^3] = 2.8328$

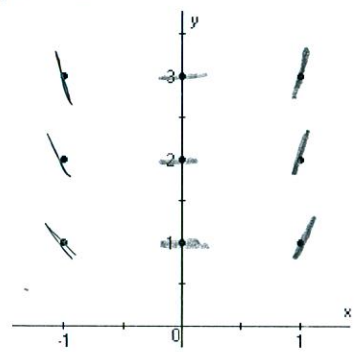
$y(1.3) = 2.8328 + .1[(2.8328)^2/(1.2)^3] = 3.2972$

$y(1.4) = 3.2972 + .1[(3.2972)^2/(1.3)^3] = 3.792$

$y(1.5) = 3.792 + .1[(3.792)^2/(1.4)^3] = 4.316$

Use the differential equation $\frac{dy}{dx} = 2xy$ to answer the following.

4. On the graph shown at the right, sketch a slope field for the differential equation at the indicated points.



$(1,1) = 2(1)(1) = 2$ $\quad (-1,1) = 2(-1)(1) = -2$

$(1,2) = 2(1)(2) = 4$ $\quad (-1,2) = 2(-1)(2) = -4$

$(1,3) = 2(1)(3) = 6$ $\quad (-1,3) = 2(-1)(3) = -6$

$(0,1) = 2(0)(1) = 0$

$(0,2) = 2(0)(2) = 0$

$(0,3) = 2(0)(3) = 0$

5. Use Euler's Method to approximate $y(1)$ given $y(0) = 1$ and $\Delta x = 0.5$. Be sure to include your calculations in a table.

$y(0) = \quad = 1$

$y(.5) = 1 + .5[2(0)(1)] = 1$

$y(1) = 1 + .5[2(.5)(1)] = 1.5$

AP Calculus- BC
Review for Additional Differentiation Test

Name________________________

6. Find the exact value for $y(1)$ given $y(0) = 1$.

x y

$\frac{dy}{dx} = 2xy$

$\frac{1}{y}dy = 2x\,dx$

$\ln|y| = \cancel{2}x^2/\cancel{2} + C$

$\ln(1) = (0)^2 + C$

$C = 0$

$e^{\ln|y|} = e^{x^2}$

$y = e^{x^2}$

$y(1) = e^{(1)^2} = e$

7. Suppose $\frac{dP}{dt} = 0.36P\left(1 - \frac{P}{1200}\right)$ and $P(0) = 200$. Find $P(t)$.

$A = 1200$
$K = .36$

$P = \frac{1200}{1 - \frac{e^{-.36t}}{C}}$

$C = \frac{200}{200 - 1200} = \frac{200}{-1000} = -\frac{1}{5}$

$P(t) = \frac{1200}{1 + 5e^{-.36t}}$

8. Solve the differential equation $\frac{dy}{dx} = 3y(y-1)$ if $y(0) = 1/4$.

$C = \frac{1/4}{1/4 - 1} = \frac{1/\cancel{4}}{-3/\cancel{4}} = -\frac{1}{3}$

$\frac{dy}{dx} = 3y(-1+y)$

$\frac{dy}{dx} = -3y(1 + \frac{y}{1})$

$A = 1 \quad K = -3$

$y(x) = \frac{1}{1 - \frac{e^{3t}}{C}}$

$y(x) = \frac{1}{1 + 3e^{3t}}$

9. The growth rate of a population P of bears in a newly established wildlife preserve is modeled by the differential equation $\frac{dP}{dt} = 0.008P(100 - P)$, where t is measure in years.

a.) What is the carrying capacity for bears in this wildlife preserve? $A = 100$

b.) Find an equation for the population of bears.

$\frac{dP}{dt} = .008P(100)\left[1 - \frac{P}{100}\right]$

$\frac{dP}{dt} = .8P\left[1 - \frac{P}{100}\right]$

K A

$P(t) = \frac{100}{1 - \frac{e^{-.8t}}{C}} = \frac{100}{1 - \frac{1}{C}e^{-.8t}}$

AP Calculus- BC
Review for Additional Differentiation Test

Name________________________

$A = 150$

10. A 2000-gallon tank can support no more than 150 guppies. Six guppies are introduced into the tank. Assume that the rate of growth of the population is $\frac{dP}{dt} = 0.0015P(\underset{150}{150} - \underset{150}{P})$ where time is in weeks.

a.) Find a formula for the guppy population in terms of t.

b.) How long will it take the guppy population to be 100? 125?

$\frac{dP}{dt} = .225P\left[1 - \frac{P}{150}\right]$

$P(t) = \frac{150}{1 - \frac{e^{-.225t}}{C}}$ $\quad C = \frac{6}{6-150} = \frac{6}{-144} = -\frac{1}{24}$

$P(t) = \frac{150}{1 + 24e^{-.225t}}$

$100 = \frac{150}{1+24e^{-.225t}}$

$1 + 24e^{-.225t} = 1.5$

$e^{-.225t} = \frac{.5}{24}$

$-.225t = \ln\left(\frac{.5}{24}\right)$

$t = \frac{\ln\left(\frac{.5}{24}\right)}{-.225}$

$t = 17.2$ days

$125 = \frac{150}{1+24e^{-.225t}}$

$\cancel{1+} 24e^{-.225t} = \underset{-1}{1.2}$

$-.225t = \ln\left(\frac{.2}{24}\right)$

$t \approx 21.3$ days

11. $\int x^2 \ln x \, dx$ $\quad u = \ln x \quad v = \frac{x^3}{3} \quad du = \frac{1}{x}dx \quad dv = x^2 dx$

$\frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^3\left(\frac{1}{x}\right)dx$

$\frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2 dx$

$\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$

12. $\int x\sin(2x)\,dx$ $\quad u = x \quad v = -\frac{1}{2}\cos 2x \quad du = dx \quad dv = \sin 2x$

$-\frac{1}{2}x\cos(2x) - \int -\frac{1}{2}\cos(2x)dx$

$-\frac{1}{2}x\cos(2x) + \frac{1}{2}\int \cos(2x)dx$

$-\frac{1}{2}x\cos(2x) + \frac{1}{4}\sin(2x) + C$

13. $\int x\sec^2 x \, dx$ $\quad u = x \quad v = \tan x \quad du = dx \quad dv = \sec^2 x\, dx$

$x\tan x - \int \tan x\, dx$

$x\tan x - \ln|\sec x| + C$

14. $\int \frac{x+4}{(x-1)(x+6)} dx$

$\frac{x+4}{(x-1)(x+6)} = \frac{A}{(x-1)} + \frac{B}{(x+6)}$

$x+4 = A(x+6) + B(x-1)$

let $x = 1$

$5 = 7A$

$A = 5/7$

let $x = -6$

$-2 = -7B$

$B = 2/7$

$\int \frac{5/7}{(x-1)} + \frac{2/7}{(x+6)}$

$\frac{5}{7}\ln|x-1| + \frac{2}{7}\ln|x+6| + C$

15. $\int \frac{2x-1}{(x-1)^2} dx$

$\frac{2x-1}{(x-1)(x-1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2}$

$2x - 1 = A(x-1) + B$

let $\cancel{x}$ $B = 0$ $\quad$ let $x = 1$

$-1 = -A + B \quad 1 = B$

$-1 = -A + 1$

$-2 = -A$

$A = 2$

$\int \frac{2}{x-1} + \frac{1}{(x-1)^2} u^2$

$2\ln|x-1| + \frac{(x-1)^{-1}}{-1} + C$

$2\ln|x-1| - \frac{1}{x-1} + C$

16. $\int \frac{1}{x^3 + x^2 - 2x} dx = \int \frac{1}{x(x-1)(x+2)}$

$\frac{1}{x(x-1)(x+2)} = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{x+2}$

$1 = A(x-1)(x+2) + Bx(x+2) + Cx(x-1)$

let $x = 0$ $\quad$ let $x = 1$ $\quad$ let $x = -2$

$1 = A(-1)(2) \quad 1 = 3B \quad 1 = 6C$

$A = \frac{1}{-2} \quad B = 1/3 \quad C = 1/6$

$\int \frac{-1/2}{x} + \frac{1/3}{x-1} + \frac{1/6}{x+2}$

$-\frac{1}{2}\ln|x| + \frac{1}{3}\ln|x-1| + \frac{1}{6}\ln|x+2| + C$

AP Calculus- BC
Review for Additional Differentiation Test

Name________________________

17. $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$

$\int_{-\infty}^{0} \frac{1}{1+x^2} + \int_{0}^{\infty} \frac{1}{1+x^2}$

$\lim_{R\to-\infty} \int_{R}^{0} \frac{1}{1+x^2} + \lim_{R\to\infty} \int_{0}^{R} \frac{1}{1+x^2}$

$\lim_{R\to-\infty} \tan^{-1}x \Big|_{R}^{0} + \lim_{R\to\infty} \int_{0}^{R} \tan^{-1}x$

$\lim_{R\to-\infty} \underbrace{\cancel{\tan^{-1}(0)}}_{0} - \tan^{-1}(-\infty) + \lim_{R\to\infty} \tan^{-1}R - \underbrace{\cancel{\tan^{-1}(0)}}_{0}$

$-\left(-\frac{\pi}{2}\right) + \frac{\pi}{2} = \pi$

18. $\int_{1}^{\infty} \frac{2dx}{x^3}$

$\lim_{R\to\infty} 2\int_{1}^{R} x^{-3}dx$

$\lim_{R\to\infty} \cancel{2}\frac{x^{-2}}{\cancel{-2}} \Big|_{1}^{R}$

$\lim_{R\to\infty} \frac{-1}{x^2} \Big|_{1}^{R}$

$\lim_{R\to\infty}$

$\cancel{\frac{-1}{R^2}} - \frac{-1}{1^2}$

1

19. $\int_{0}^{\boxed{1}} \frac{dx}{(x-1)^{2/3}}$ ud

$\lim_{R\to1^-} \int_{0}^{R} \frac{1}{u^{2/3}} du$

$\lim_{R\to1^-} 3u^{1/3} \Big|_{0}^{R}$

$\lim_{R\to1^-} 3u^{1/3} - 3(0)^{1/3}$

$\lim_{R\to1^-} 3(x-1)^{1/3} - 3(x-1)^{1/3}$

$\lim_{R\to1^-} 3(R-1)^{1/3} - 3(0-1)^{1/3}$

$= 3$

20. $\int \sin^4 x\, dx$

$-\frac{1}{4}\cos^3 x \sin x + \frac{3}{4}\int \sin^2 x$

$-\frac{1}{4}\cos^3 x \sin x + \frac{3}{4}\left[\frac{-1}{2}\sin x\cos x + \frac{1}{2}\int \sin^0 x\right.$

$-\frac{1}{4}\cos^3 x \sin x - \frac{3}{8}\sin x\cos x + \frac{3}{4}x + C$

21. $\int_{\pi/2}^{3\pi/4} \sin^5 x \cos^2 x\, dx$

$\sin x \cdot \sin^2 x \cdot \sin^2 x \cdot \cos^2 x$

$u = \cos x$
$du = -\sin x$

$-\int_{\pi/2}^{3\pi/2} -\sin x(1-\cos^2 x)(1-\cos^2 x)\cos^2 x$

$-\int (1-u^2)(1-u^2)u^2\, du$

$-\int (1-2u^2+u^4)u^2 du$

$-\int u^2 - 2u^4 + u^6\, du$

$-\frac{u^3}{3} + \frac{2u^5}{5} - \frac{u^7}{7} + C$

$-\frac{1}{3}\cos^3 + \frac{2}{5}\cos^5 x - \frac{1}{7}\cos^7 x + C$

22. $\int \tan^3 x \sec x\, dx$

$\int \tan^2 x \cdot \tan x \cdot \sec x\, dx$

$\int (1-\sec^2 x)\tan x \cdot \sec x$

$u = \sec x$
$du = \sec x \tan x$

$\int 1 - u^2\, du$

$u - \frac{u^3}{3} + C$

$\sec x - \frac{1}{3}\sec^3 x + C$