Name ______ **BC Calculus – Polar AP Practice**

You may use a calculator for the questions 1-2.

1. A polar curve is given by $r = \dfrac{3}{2-\cos\theta}$. The slope of the curve at $\theta = \dfrac{\pi}{2}$ is

$x = \dfrac{3}{2-\cos\theta} \cdot \cos\theta$

$y = \dfrac{3}{2-\cos\theta} \sin\theta$

A. 0 B. 0.5 C. 0.75

D. -0.75 E. not defined

$\dfrac{dy}{dx} = \dfrac{(2-\cos\theta)(3\cos\theta) - 3\sin^2\theta}{(2-\cos\theta)(-3\sin\theta) - 3\cos\theta\sin\theta}$

$\dfrac{(2)(0)-3}{(2)(-3)-0} = \dfrac{-3}{-6}$

2. The area inside the polar curve r = 3 + 2cosθ is

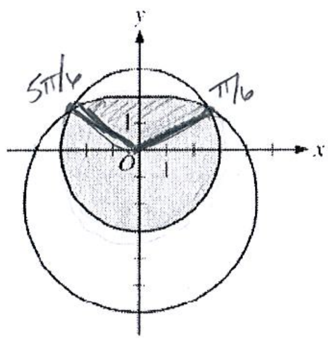
A. 9.425 B. 18.850 C. 28.274

D. 34.558 E. 69.115

$\frac{1}{2}\int_0^{2\pi} (3+2\cos\theta)^2 \, d\theta$

3. Calculator Free Response

The graphs of the polar curves r = 3 and r = 4 – 2sinθ are shown in the figure below. The curves intersect when $\theta = \dfrac{\pi}{6}$ and $\theta = \dfrac{5\pi}{6}$.



a. Let S be the shaded region that is inside the graph of r = 3 and also inside the graph of r = 4 – 2sinθ. Find the area of S.

$\frac{1}{2}\int_{\pi/6}^{5\pi/6} (4-2\sin\theta)^2 \, d\theta + \frac{1}{2}\int_{5\pi/6}^{13\pi/6} 3^2 \, d\theta = 24.709$

b. A particle moves along the polar curve r = 4 – 2sinθ so that at time t seconds, $\theta = t^2$. Find the time t in the interval 1 < t < 2 for which the x-coordinate of the particle's position is -1.

$x = r\cos\theta$

$x(\theta) = (4-2\sin\theta)\cos\theta$

$x(t) = (4-2\sin(t^2))\cos(t^2)$

$(4-2\sin(t^2))\cos(t^2) = -1$

$\boxed{t = 1.428}$

c. For the particle described in part b, find the position vector in terms of t. Find the velocity vector at time t = 1.5.

$y = r\sin\theta$

$y(\theta) = (4-2\sin\theta)\sin\theta$

$y(t) = (4-2\sin t^2)\sin t^2$

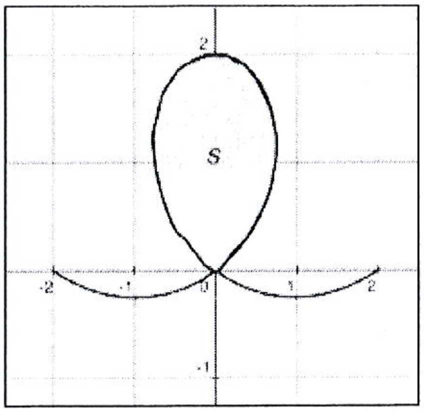
Position $= \langle (4-2\sin(t^2))\cos(t^2), (4-2\sin(t^2))\sin(t^2) \rangle$

velocity $= \langle x'(t), y'(t) \rangle \big|_{t=1.5}$

$= \langle -8.072, -1.673 \rangle$

4. Calculator Free Response

The graph of the polar curve r = 2 + 4 sin θ for $\pi \le \theta \le 2\pi$ is shown in the diagram below. Let S be the shaded region bounded by the curve above the x-axis.



a. Write an integral expression for the area of S.

$$S = \frac{1}{2}\int_{7\pi/6}^{11\pi/6} (2+4\sin\theta)^2\, d\theta$$

$2+4\sin\theta = 0$

$\sin\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

b. Write an expression for $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$ in terms of θ.

$X = (2+4\sin\theta)\cos\theta = 2\cos\theta + 4\sin\theta\cos\theta$

$\frac{dx}{d\theta} = -2\sin\theta - 4\sin^2\theta + 4\cos^2\theta$

$y = (2+4\sin\theta)\sin\theta = 2\sin\theta + 4\sin^2\theta$

$\frac{dy}{d\theta} = 2\cos\theta + 8\sin\theta\cos\theta$

c. Write an expression for the arc length of the curve from $\pi \le \theta \le 2\pi$.

$$\int_{\pi}^{2\pi} \sqrt{(-2\sin\theta - 4\sin^2\theta + 4\cos^2\theta)^2 + (2\cos\theta + 8\sin\theta\cos\theta)^2}\, d\theta$$

d. Write an equation in terms of x and y for the line tangent to the graph of the polar curve at the point where $\theta = \frac{5\pi}{4}$ I

$X\left(\frac{5\pi}{4}\right) = 2-\sqrt{2}$

$y\left(\frac{5\pi}{4}\right) = 2-\sqrt{2}$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \left.\frac{2\cos\theta + 8\sin\theta\cos\theta}{-2\sin\theta - 4\sin^2\theta + 4\cos^2\theta}\right|_{\theta=\frac{5\pi}{4}} = 2\sqrt{2}-1$$

$$y-(2-\sqrt{2}) = (2\sqrt{2}-1)[x-(2-\sqrt{2})]$$