Name BC Calculus – Parametric/Vector AP Practice

No Calculators are to be used for Questions 1 – 4.

1. A particle moves in the x-y plane such that its position for time $t \geq 0$ is given by $x(t) = 3t^2 - 19t$ and $y(t) = e^{2t-7}$. What is the slope of the tangent to the path of the particle when $t = 4$?

A. $-\frac{e}{28}$ B. $-\frac{28}{e}$ C. $\frac{e}{5}$

D. $\frac{2e}{5}$ E. $\frac{5}{2e}$

$\frac{dy}{dx} = \frac{2e^{2t-7}}{6t-19}\Big|_{t=4}$

$\frac{2e}{5}$

2. The path of a particle in the x-y plane is given by the parametric equations $x(t) = \ln t$ and $y(t) = 5t^2 + 11$ for $t > 0$. An integral expression that represents the length of the path from $t = 2$ to $t = 6$ is

A. $\int_2^6 \sqrt{\frac{1}{t^2} + 100t^2}\, dt$ B. $\int_2^6 \sqrt{(\ln t)^2 + (5t^2+11)^2}\, dt$ C. $\int_2^6 \left|5t^2 + 11 - \ln t\right| dt$

D. $\int_2^6 \sqrt{1 + \frac{1}{t^2}}\, dt$ E. $\int_2^6 \sqrt{1 + 100t^2}\, dt$

$\int_2^6 \sqrt{\left(\frac{1}{t}\right)^2 + (10t)^2}$

3. A plane curve has parametric equations $x(t) = t^2$ and $y(t) = t^4 + 3t^2$. An expression for the rate of change of the slope of the tangent to the path of the curve is

A. $2t^2 + 3$ B. $4t$ C. $6t^2 + 3$

D. $t^2 + 3$ E. 2

$\frac{dy}{dx} = \frac{4t^3 + 6t}{2t} = 2t^2 + 3$

$\frac{d^2y}{dx^2} = 4t \cdot \frac{1}{2t} = 2$

4. The position of a particle moving in the xy-plane is given by the parametric equations $x = t^3 - 3t^2$ and $y = 2t^3 - 3t^2 - 12t$. For what values of t is the particle at rest?

A. -1 only B. 0 only C. 2 only

D. -1 and 2 only E. -1, 0, and 2

$3t^2 - 6t = 0$
$3t(t-2) = 0$
$t = 0 \quad t = 2$

$6t^2 - 6t - 12 = 0$
$t^2 - t - 2 = 0$
$(t-2)(t+1) = 0$
$t = 2 \quad t = -1$

5. Calculator Free Response

The velocity vector of a particle moving in the plane has components given by

$$\frac{dx}{dt} = 14\cos(t^2)\sin(e^t) \text{ and } \frac{dy}{dt} = 1 + 2\sin(t^2) \text{ , for } 0 \le t \le 1.5.$$

At time t = 0, the position of the particle is (-2, 3).

a. For 0 < t < 1.5, find all values of t at which the line tangent to the path of the particle is vertical.

$\frac{dx}{dt} = 0 \qquad 14\cos(t^2)\sin(e^t) = 0$

$t = 1.253 \quad t = 1.445$

b. Write an equation for the line tangent to the path of the particle at t = 1.

$\frac{dy}{dx} = \left.\frac{1+2\sin(t^2)}{14\cos(t^2)\sin(e^t)}\right|_{t=1} = \frac{1+2\sin(1)}{14\cos(1)\sin(e)} = .863$

$x(1) = -2 + \int_0^1 \frac{dx}{dt}\,dt$

$y(1) = 3 + \int_0^1 \frac{dy}{dt}\,dt$

$(9.315, 4.621)$

$y - 4.621 = .863(x - 9.315)$

c. Find the speed of the particle at t = 1.

$\sqrt{(x'(1))^2 + (y'(1))^2} = 4.105$

d. Find the acceleration vector of the particle at t = 1.

$\langle x''(1), y''(1) \rangle = \langle -28.425, 2.161 \rangle$

6. Calculator Free Response

A particle moving along a curve in the xy-plane has position (x(t), y(t)) at time t ≥ 0 with

$$\frac{dx}{dt} = 4e^{-\frac{t}{2}} \text{ and } \frac{dy}{dt} = -4\sin\left(\frac{t^3}{3}\right)$$

The particle is at postion (2, -3) at t = 3.

a. Find the acceleration vector at time t = 3.

$\langle x''(3), y''(3) \rangle = \langle -.446, 32.8 \rangle$

b. Find the y-coordinate of the postion of the particle at time t = 0.

$-3 - \int_0^3 -4\sin\left(\frac{t^3}{3}\right) dt = -.037$

c. On the interval 0 < t < 3, how many times is the speed of the particle equalt to 3?

$\sqrt{\left(4e^{-t/2}\right)^2 + \left(-4\sin\left(\frac{t^3}{3}\right)\right)^2} = 0$ 7 times (graph y_1 = speed and $y_2 = 3$).

d. Find the total distance traveled by the particle over the interval 0 ≤ t ≤ 3.

$\int_0^3 \sqrt{\left(4e^{-t/2}\right)^2 + \left(-4\sin(t^3/3)\right)^2}\,dt = 9.591$

. Calculator Free Response

A particle moves along a curve so that its position at time t is (x(t), y(t)) where $x(t) = t^2 - 3t + 9$ and y(t) is not specifically given. Both x and y are measured in inches and t is measured in seconds. It is known that $\frac{dy}{dt} = t^2 e^{t-2} - 2$.

a. Find the speed of the particle at time t = 2 seconds.

$$\text{Speed} = \sqrt{(2t-3)^2 + (t^2 e^{t-2} - 2)^2}\Big|_{t=2} = \sqrt{5} = 2.236$$

b. For $0 \le t \le 3$, find the total distance traveled by the particle.

$$\int_0^3 \sqrt{(2t-3)^2 + (t^2 e^{t-2} - 2)^2}\, dt = 13.717$$

c. Find the time t, $0 \le t \le 3$ when the line tangent to the path of the particle is horizontal. Is the particle moving left or right at that time? Give a reason for your answer.

$$\frac{dy}{dt} = 0 \qquad t^2 e^{t-2} - 2 = 0 \qquad t = 1.669$$

$$\frac{dx}{dt}\Big|_{t=1.669} = .338 > 0$$

$\therefore$ particle is moving right

d. There is a point with x-coordinate 7 through which the particle passes twice. Find each of the following:

i. The two values of t when that occurs.

$$x(t) = 7$$
$$t^2 - 3t + 9 = 7$$
$$t = 1,\ t = 2$$

ii. The slopes of the lines tangent to the particle's path at that point.

$$\frac{dy}{dx}\Big|_{t=1} = 1.632 \qquad \frac{dy}{dx}\Big|_{t=2} = 2$$

iii. The y-coordinate given that y(3) = 5e – 7.

$$5e - 7 - \int_2^3 (t^2 e^{t-2} - 2)\, dt = -3$$

$$5e - 7 - \int_1^3 (t^2 e^{t-2} - 2)\, dt = -3$$