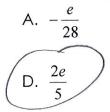
## No Calculators are to be used for Questions 1-4.

1. A particle moves in the x-y plane such that its position for time  $t \ge 0$  is given by  $x(t) = 3t^2 - 19t$  and  $y(t) = e^{2t-7}$ . What is the slope of the tangent to the path of the particle when t = 4?



$$B. -\frac{28}{e}$$

E. 
$$\frac{5}{2e}$$

C. 
$$\frac{e}{5}$$

$$\frac{dy}{dx} = \frac{2e^{2t-7}}{6t-19} \Big|_{t=4}$$

$$\frac{2e}{5}$$

2. The path of a particle in the x-y plane is given by the parametric equations  $x(t) = \ln t$  and  $y(t)=5t^2+11$  for t>0. An integral expression that represents the length of the path from t=2 to t = 6 is

$$A. \int_{2}^{6} \sqrt{\frac{1}{t^{2}} + 100t^{2}} dt$$

B. 
$$\int_{2}^{6} \sqrt{(\ln t)^{2} + (5t^{2} + 11)^{2}} dt$$
 C.  $\int_{2}^{6} |5t^{2} + 11 - \ln t| dt$ 

C. 
$$\int_{2}^{6} |5t^2 + 11 - \ln t| dt$$

$$D. \int_{2}^{6} \sqrt{1 + \frac{1}{t^2}} dt$$

$$E. \int_{2}^{6} \sqrt{1 + 100t^2} \, dt$$

$$\int \int (\frac{1}{4})^{2} + (b\ell)^{2}$$

3. A plane curve has parametric equations  $x(t) = t^2$  and  $y(t) = t^4 + 3t^2$ . An expression for the rate of change of the slope of the tangent to the path of the curve is

A. 
$$2t^2 + 3$$

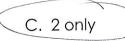
D. 
$$t^2 + 3$$

C. 
$$6t^2 + 3$$

$$\frac{dy}{dx} = \frac{4t^3 + 6t}{2t} = 2t^2 + 3$$

$$\frac{d^2y}{dx^2} = 4t \cdot \frac{1}{2t} = 2$$

4. The position of a particle moving in the xy-plane is given by the parametric equations x =  $t^3 - 3t^2$  and  $y = 2t^3 - 3t^2 - 12t$ . For what values of t is the particle at rest?



$$3t^2-6t=0$$
  $(6t^2-6t-12=0)$   
 $3t(t-2)=0$   $t^2-t-2=0$   
 $t=0$   $t=2$   $(t-2)(t=10=0)$   
 $t=2$   $t=-1$ 

## Calculator Free Response

The velocity vector of a particle moving in the plane has components given by

$$\frac{dx}{dt} = 14\cos(t^2)\sin(e^t) \text{ and } \frac{dy}{dt} = 1 + 2\sin(t^2) \text{ , for } 0 \le t \le 1.5.$$

At time t = 0, the position of the particle is (-2, 3).

a. For 0 < t < 1.5, find all values of t at which the line tangent to the path of the particle is vertical.

$$\frac{dx}{dt} = 0$$
  $\frac{14\cos(t^2)\sin(e^t)}{t} = 0$   
 $\frac{dx}{dt} = 0$   $\frac{dx}{dt} = 0$ 

b. Write an equation for the line tangent to the path of the particle at t = 1.  $\frac{dy}{dx} = \frac{1+2\sin(t^2)}{14\cos(t^2)\sin(e^4)} \Big|_{t=1} = \frac{1+2\sin(t)}{14\cos(t)\sin(e)} = .863 \quad x(1)=-2+\int \frac{dx}{dt} dt$ c. Find the speed of the particle at t = 1.  $y - 4.621 = .863(x-9.315)y(1)=3+\int \frac{dy}{dt} dt$ 

c. Find the speed of the particle at 
$$t = 1$$
.  $y - 4.621 = .863(x - 9.315)y^{(1)} = 3 + \int_{0.007}^{0.007} dt$   $\sqrt{(\chi'(1))^2 + (y'(1))^2} = 4.105$  (9.315,4.621)

d. Find the acceleration vector of the particle at t = 1.

$$\langle x''(1), y''(1) \rangle = \langle -28.425, 2.161 \rangle$$

## 6. Calculator Free Response

A particle moving along a curve in the xy-plane has position (x(t), y(t)) at time t > 0 with

$$\frac{dx}{dt} = 4e^{-\frac{t}{2}}$$
 and  $\frac{dy}{dt} = -4\sin\left(\frac{t^3}{3}\right)$ 

The particle is at postion (2, -3) at t = 3.

a. Find the acceleration vector at time t = 3.

b. Find the y-coordinate of the postion of the particle at time t = 0.

$$-3-\int_{-4\sin(\frac{t^3}{3})}^{2}dt = -.037$$

c. On the interval  $0^{\circ}$ < t < 3, how many times is the speed of the particle equalt to

$$\sqrt{(4e^{-t/2})^2 + (-4\sin(\frac{t^3}{3}))^2} = 0$$
Thinks (90 april (91 = speed and 10 3).

d. Find the total distance traveled by the particle over the interval  $0 \le t \le 3$ .

## . Calculator Free Response

A particle moves along a curve so that its position at time t is (x(t), y(t)) where  $x(t) = t^2 - 3t + 9$  and y(t) is not specifically given. Both x and y are measured in inches and t is measured in seconds. It is known that  $\frac{dy}{dt} = t^2 e^{t-2} - 2$ .

a. Find the speed of the particle at time t = 2 seconds.

Speed = 
$$\sqrt{(2t-3)^2 + (t^2e^{t-2}-2)^2}$$
 |  $t=2=15=2.236$ 

b. For  $0 \le t \le 3$ , find the total distance traveled by the particle.

$$\int_{0}^{3} \sqrt{(2t-3)^{2}+(t^{2}e^{t-2}-2)^{2}} dt = 13.717$$

c. Find the time t,  $0 \le t \le 3$  when the line tangent to the path of the particle is horizontal. Is the particle moving left or right at that time? Give a reason for your answer.

ver.  

$$\frac{dx}{dt} = 0$$
  $\frac{dx}{dt}|_{t=1.669} = .338>0$   
 $\frac{dx}{dt}|_{t=1.669} = .338>0$   
i. particle is moving right

- d. There is a point with x-coordinate 7 through which the particle passes twice. Find each of the following:
  - i. The two values of t when that occurs.

$$X(t)=7$$
  $t=1, t=2$   $t^2-3t+9=7$ 

ii. The slopes of the lines tangent to the particle's path at that point.

$$\frac{dy}{dx}\Big|_{t=1} = 1.632$$
  $\frac{dy}{dx}\Big|_{t=2} = 2$ 

iii. The y-coordinate given that y(3) = 5e - 7.

$$5e-7-\int_{2}^{3}(t^{2}e^{t-2}-2)dt=-3$$