Name **Multiple choice Review- Derivatives (2)**

1. Let $f(x)=4x^3-3x-1$. An equation of the line tangent to $y=f(x)$ at $x=2$ is

A. $y=25x-5$

B. $y=45x+65$

C. $y=45x-65$

D. $y=65-45x$

E. $y=65x-45$

$f'(x)=12x^2-3\big|_{x=2}=12(4)-3=45$

$f(2)=4(2)^3-3(2)-1=32-6-1=25$

$y-25=45(x-2)$

$y-25=45x-90$

$y=45x-65$

2. $\dfrac{d}{dx}\sec^{-1}\left(x^2\right)=\dfrac{1}{x^2\sqrt{(x^2)^2-1}}\cdot 2x=\dfrac{2}{x\sqrt{x^4-1}}$

A. $\dfrac{2}{x\sqrt{x^4-1}}$

B. $\dfrac{2}{x\sqrt{x^2-1}}$

C. $\dfrac{2}{x\sqrt{1-x^4}}$

D. $\dfrac{2}{x\sqrt{1-x^2}}$

E. $\dfrac{2x}{\sqrt{1-x^4}}$

3. If $g(x)=\dfrac{x-2}{x+2}$, then $g'(2)=$

A. 1

B. -1

C. $\dfrac{1}{4}$

D. $-\dfrac{1}{4}$

E. 0

$g'(x)=\dfrac{(x+2)(1)-(x-2)(1)}{(x+2)^2}$

$=\dfrac{x+2-x+2}{(x+2)^2}=\dfrac{4}{(x+2)^2}\Big|_{x=2}=\dfrac{4}{4^2}=\dfrac{1}{4}$

4. If $y=\dfrac{3}{4+x^2}$, then $\dfrac{dy}{dx}=$

$y=3(4+x^2)^{-1}$

$y'=-3(4+x^2)^{-2}\cdot 2x$

$=\dfrac{-6x}{(4+x^2)^2}$

A. $\dfrac{3}{2x}$

B. $\dfrac{3x}{\left(1+x^2\right)^2}$

C. $\dfrac{6x}{\left(4+x^2\right)^2}$

D. $\dfrac{-6x}{\left(4+x^2\right)^2}$

E. $\dfrac{-3}{\left(4+x^2\right)^2}$

$y=3(4+x^2)^{-1}$

5. If $x+2xy-y^2=2$, then at the point (1, 1), $\dfrac{dy}{dx}=$

A. $\dfrac{3}{2}$

B. $\dfrac{1}{2}$

C. 0

D. $-\dfrac{3}{2}$

E. nonexistent

$1+2xy'+y\cdot 2-2yy'=0$

$2xy'-2yy'=-1-2y$

$y'(2x-2y)=-1-2y$

$y'=\dfrac{-1-2y}{2x-2y}\Big|_{(1,1)}=\dfrac{-1-2}{2-2}=\dfrac{-3}{0}$

6. The derivative of $\sqrt{x}-\dfrac{1}{x\sqrt[3]{x}}$ is

$=x^{1/2}-\dfrac{1}{x\cdot x^{1/3}}=x^{1/2}-\dfrac{1}{x^{4/3}}$

$=x^{1/2}-x^{-4/3}$

$y'=\frac{1}{2}x^{-1/2}+\frac{4}{3}x^{-7/3}$

A. $\frac{1}{2}x^{-1/2}-x^{-4/3}$

B. $\frac{1}{2}x^{-1/2}+\frac{4}{3}x^{-7/3}$

C. $\frac{1}{2}x^{-1/2}-\frac{4}{3}x^{-1/3}$

D. $-\frac{1}{2}x^{-1/2}+\frac{4}{3}x^{-7/3}$

E. $-\frac{1}{2}x^{-1/2}-\frac{4}{3}x^{-1/3}$

7. An object moves along the x-axis so that at time t ($t>0$) its position is given by $x(t)=t^4+t^3-30t^2+88t$ At the instant when the acceleration becomes zero, the velocity of the object is approximately

A. 244

B. 12

C. 0

D. -12

E. -24

$v(t)=x'(t)=4t^3+3t^2-60t+88$

$v(2)=4(8)+3(4)-60(2)+88$
$=-12$

$a(t)=x''(t)=12t^2+6t-60$
$0=12t^2+6t-60$ $t>0$
$0=6(2t+5)(t-2)$ $\therefore t=2$

8. If the position of a particle on the x-axis at time t is $-5t^2$, then the average velocity of the particle for [0, 3]

Slope!

A. -45

B. -30

C. -15

D. -10

E. -5

Find y-coordinates
$-5(0)^2=0$
$\therefore (0,0)$
$-5(3)^2=-45$
$\therefore (3,-45)$

$\frac{-45-0}{3-0}=\frac{-45}{3}$
$=-15$

9. The slope of the line tangent to the graph of $\ln(xy)=x$ at the point where x = 1 is

$\ln 1y=1$
find y
$e^{\ln y}=e^1$
$y=e$

A. 0

B. 1

C. e

D. e^2

E. 1 − e

$\frac{d}{dx}(\ln(xy)=x)$

$\frac{1}{xy}(xy'+y)=1$

$xy'+y=xy$
$xy'=xy-y$
$y'=\frac{xy-y}{x}\Big|_{(1,e)}$

$\frac{e-e}{1}=0$

10. $\lim_{h\to 0}\left(\frac{\cos(x+h)-\cos x}{h}\right)=$

$\leftarrow f(x)=\cos x$
$\therefore f'(x)=-\sin x$

A. $\sin x$

B. $-\sin x$

C. $\cos x$

D. $-\cos x$

E. Does not exist

11. If $y=\cos^2 x-\sin^2 x$, then $y'=$

$y=(\cos x)^2-(\sin x)^2$

A. -1

B. 0

C. $-2(\cos x+\sin x)$

D. $2(\cos x+\sin x)$

E. $-4(\cos x)(\sin x)$

$y'=2\cos x(-\sin x)-2\sin x\cdot\cos x$
$=-2\cos x\sin x-2\cos x\sin x$
$=-4\cos x\sin x$

12. $\frac{d}{dx}\arcsin\left(\frac{x}{2}\right)$

$\frac{1}{\sqrt{1-(\frac{x}{2})^2}}\cdot\frac{1}{2}=\frac{1}{2\sqrt{1-\frac{x^2}{4}}}$

$(\frac{4}{4})$

$=\frac{1}{2\sqrt{\frac{4-x^2}{4}}}$

$=\frac{1}{\frac{2\sqrt{4-x^2}}{\sqrt{4}}}$

$=\frac{1}{\frac{\cancel{2}\sqrt{4-x^2}}{\cancel{2}}}$

$=\frac{1}{\sqrt{4-x^2}}$

A. $-\frac{2}{\sqrt{4-x^2}}$

B. $-\frac{1}{\sqrt{4-x^2}}$

C. $\frac{2}{4+x^2}$

D. $\frac{2}{\sqrt{4-x^2}}$

E. $\frac{1}{\sqrt{4-x^2}}$

13. An equation for a tangent to the graph of $y=\arctan\frac{x}{3}$ at the origin is:

$(0,0)$

A. $x-3y=0$

B. $x-y=0$

C. $x=0$

D. $y=0$

E. $3x-y=0$

$y'=\frac{1}{(\frac{x}{3})^2+1}\cdot\frac{1}{3}=\frac{1}{(\frac{x^2}{9}+1)3}\Big|_{(0,0)}=\frac{1}{(0+1)3}=\frac{1}{3}$

$y-0=\frac{1}{3}(x-0)$ $y=\frac{1}{3}x$ $3y=x$ $x-3y=0$

14. A particle moves along the x-axis so that at any time $t>0$ its position is given by $x(t)=t^3-3t^2-9t+1$. For what values of t is the particle at rest?

$v(t)=0$

A. None

B. 1 only

C. 3 only

D. 5 only

E. 1 and 3

$v(t)=3t^2-6t-9$
$0=3t^2-6t-9$
$0=3(t^2-2t-3)$
$0=3(t-3)(t+1)$

$t=3$
$t=-1$
$t>0$

$\ln e^{3x} = 3x \ln e$ and $\ln e = 1$:)

15. $\frac{d}{dx}\left(\ln e^{3x}\right) = \frac{d}{dx}(3x) =$ 3

A. 1

B. 3

C. $3x$

D. $\frac{1}{e^{3x}}$

E. $\frac{3}{e^{3x}}$

16. Find $\frac{dy}{dx}$ for $e^y = xy$

$e^y \cdot y' = xy' + y$

$e^y y' - xy' = y$

$y'(e^y - x) = y$

$y' = \frac{y}{e^y - x}$

A. $\ln x + \ln y$

B. $\frac{x+y}{xy}$

C. $\frac{xy}{x+y}$

D. $\frac{xy-x}{y}$

hmm... no e^y

well, $e^y = xy$

$\therefore y' = \frac{y}{xy-x}$

E. $\frac{y}{xy-x}$

17. If $\tan(x+y) = x$, then $\frac{dy}{dx} =$

A. $\tan^2(x+y)$

B. $\sec^2(x+y)$

C. $\ln|\sec(x+y)|$

D. $\sin^2(x+y) - 1$

E. $\cos^2(x+y) - 1$

$\sec^2(x+y)(1+y') = 1$

$1 + y' = \frac{1}{\sec^2(x+y)}$

$y' = \frac{1}{\sec^2(x+y)} - 1$

18. $\frac{d}{dx}[Arc\tan(3x)] =$ $\frac{1}{1+(3x)^2} \cdot 3 = \frac{3}{1+9x^2}$

A. $\frac{1}{1+9x^2}$

B. $\frac{3}{1+9x^2}$

C. $\frac{3}{\sqrt{4x^2-1}}$

D. $\frac{3}{1+3x}$

E. None of the above

19. If $f(x) = e^{2x}$ and $g(x) = \ln x$, then the derivative of $y = f(g(x))$ at $x = e$ is

A. e^2

B. $2e^2$

C. $2e$

D. 2

E. Undefined

$f(g(x)) = f(\ln x) = e^{2\ln x}$

$e^{2\ln x} = e^{\ln x^2} = x^2$

$\frac{d}{dx}(x^2) = 2x|_{x=e} = 2e$

20. If $x^2 + 2xy - 3y = 3$, then the value of $\frac{dy}{dx}$ at $x = 2$ is need y

$2^2 + 2(2)y - 3y = 3$

$4 + 4y - 3y = 3$

$4 + y = 3$

$y = -1$

A. 1

B. 2

C. -2

D. $\frac{10}{3}$

E. $-\frac{1}{2}$

$2x + 2xy' + 2y - 3y' = 0$

$2xy' - 3y' = -2x - 2y$

$y'(2x-3) = -2x - 2y$

$y' = -\frac{2x-2y}{2x-3}\Big|_{(2,-1)} = \frac{-2}{1}$

21. If $h(x) = \left(x^2 - 4\right)^{3/4} + 1$, then the value of $h'(2)$ is

A. 3

B. 2

C. 1

D. 0

E. does not exist

$h'(x) = \frac{3}{4}(x^2-4)^{-1/4}(2x)$

$h'(2) = \frac{3}{4}(2^2-4)^{-1/4}(4)$

$= \frac{3}{4\sqrt[4]{0}} \cdot 4 = \frac{12}{0}$

undefined slope

22. If $(x-y)^2 = y^2 - xy$, then $\frac{dy}{dx} =$

A. $\frac{2x-y}{2y-x}$

B. $\frac{2x-y}{2x}$

C. $\frac{2x-y}{x}$

D. $\frac{2x+3y}{x}$

E. undefined

$2(x-y)(1-y') = 2yy' - [xy' + y]$

$(2x-2y)(1-y') = 2yy' - xy' - y$

$2x - 2xy' - 2y + 2yy' = 2yy' - xy' - y$

$-2xy' + 2yy' - \cancel{2yy'} + xy' = -2x + 2y - y$

$-xy' = -2x + y$ $y' = \frac{-2x+y}{-x}$

$\frac{-1}{1+(3x)^2} \cdot 3 = \frac{-3}{1+9x^2}$

23. Consider the curve $x + xy + 2y^2 = 6$. The slope of the line tangent to the curve at the point (2, 1) is

A. $\frac{2}{3}$

B. $\frac{1}{3}$

C. $-\frac{1}{3}$

D. $-\frac{1}{5}$

E. $-\frac{3}{4}$

$1 + xy' + y + 4yy' = 0$

$xy' + 4yy' = -1 - y$

$y'(x+4y) = -1 - y$

$y' = \frac{-1-y}{x+4y}\Big|_{(2,1)} = \frac{-1-1}{2+4} = -\frac{2}{6} = -\frac{1}{3}$

24. $\frac{d}{dx}\cot^{-1}(3x)$

A. $-\frac{3}{1+9x^2}$

B. $-\frac{1}{1+9x^2}$

C. $\frac{1}{1+9x^2}$

D. $\frac{3}{1+9x^2}$

E. $\frac{3}{\sqrt{1-9x^2}}$

25. Let $f(x) = \frac{\ln e^{2x}}{x-1} = \frac{2x}{x-1}$ for $x > 1$. If g is the inverse of f, then $g'(3) = \frac{d}{dx}\left[f^{-1}(x)\right]\Big|_3$

A. 2

B. 1

C. 0

D. -1

E. -2

$\frac{1}{f'(f^{-1}(3))} = \frac{1}{f'(3)} = \frac{1}{-\frac{1}{2}} = -2$

$3 = \frac{2x}{x-1}$

$3x - 3 = 2x$

$-3 = -x$

$3 = x$

$f'(x) = \frac{(x-1)\cdot 2 - 2x}{(x-1)^2} = \frac{2x-2-2x}{(x-1)^2} = \frac{-2}{(x-1)^2}$

$f'(3) = \frac{-2}{4}$

26. If $y = x^{\ln x}$, then $y' =$

A. $\frac{x^{\ln x}\ln x}{x^2}$

B. $x^{1/x}\ln x$

C. $\frac{2x^{\ln x}\ln x}{x}$

D. $\frac{x^{\ln x}\ln x}{x}$

E. None of the above

$\ln y = \ln x^{\ln x}$

$\ln y = \ln x \cdot \ln x$

$\frac{1}{y}y' = \ln x \cdot \frac{1}{x} + \frac{1}{x}\ln x$

$\frac{1}{y}y' = \frac{\ln x}{x} + \frac{\ln x}{x}$

$\frac{1}{y}y' = \frac{2\ln x}{x}$

$y' = \frac{2\ln x}{x} \cdot y$

$y' = \frac{2\ln x}{x} \cdot x^{\ln x}$

27. If $\tan(xy) = x$, then $\frac{dy}{dx} =$

A. $\frac{1 - y\tan(xy)\sec(xy)}{x\tan(xy)\sec(xy)}$

B. $\frac{\sec^2(xy) - y}{x}$

C. $\cos^2(xy)$

D. $\frac{\cos^2(xy)}{x}$

E. $\frac{\cos^2(xy) - y}{x}$

$\sec^2(xy)[xy' + y] = 1$

$[xy' + y] = \frac{1}{\sec^2(xy)}$

$[xy' + y] = \cos^2(xy)$

$xy' = \cos^2(xy) - y$

$y' = \frac{\cos^2(xy) - y}{x}$

28. If $y^2 - 3x = 7$, then $\frac{d^2y}{dx^2} =$

A. $\frac{-6}{7y^3}$

B. $\frac{-3}{7y^3}$

C. 3

D. $\frac{3}{2y}$

E. $\frac{-9}{4y^3}$

$2yy' - 3 = 0$

$2yy' = 3$

$y' = \frac{3}{2y}$

$y'' = \frac{2y(0) - 3(2y')}{(2y)^2}$

$= \frac{-6y'}{4y^2} = \frac{-6\left(\frac{3}{2y}\right)}{4y^2} = \frac{\frac{-18}{2y}}{4y^2}$

$= \frac{-18}{8y^3} = \frac{-9}{4y^3}$