$G(t)$ = unprocessed gravel arrives = $\frac{\text{tons}}{\text{hr}}$ $t=0$ plant has 500 tons process gravel = 100 $\frac{\text{tons}}{\text{hour}}$

Calculator
Rates in Context #3

Name________________
Date__________Period__

On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday ($t = 0$), the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

a.) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

$Y_1 = 90 + 45\cos\left(\frac{x^2}{18}\right)$ $G'(5) = -24.5875$ $\frac{\text{tons}/\text{hr}}{\text{hr}} = \frac{\text{tons}}{\text{hr}^2}$

graph
2nd calc
6: dy/dx
5

+1: answer

The rate gravel arrives is decreasing by 24.5875 tons/hour^2 at the 5th hour

+1: Interpretation with units.

b.) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

$\int_0^8 G(t)\,dt = 825.551$

+1: Integral +1: answer

c.) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

Compare: Rate arrive $G(5)$ 98.141 $\frac{\text{tons}}{\text{hour}}$ Rate processed 100 100 $\frac{\text{tons}}{\text{hour}}$

+1: Comparing $G(5)$ & 100

Decreasing because the rate gravel is being processed is greater than the rate arriving. +1: Conclusion

d.) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

Amount gravel $= 5\cancel{0}\cancel{0} + \int_0^t G(t) - 100\,dt$

Amount$' = G(t) - 100$

$0 = G(t) - 100$ +1: considers $A'(t) = 0$

$t = 4.923$

t	Amount
0	500
$t = 4.923$	$500 + \int_0^{4.923} G(t) - 100 = 635.376$
8	$500 + \int_0^8 G(t) - 100\,dt = 525.551$

+1: Justification

+1: answer Maximum is 635.376 tons.

Non-Calculator
Rates in Context #5

$\frac{dW}{dt} = \frac{\text{tons}}{\text{year}}$ waste $W(0) = 1400$ tons

Name________________
Date___________Period___

At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.

a.) Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 $\left(\text{time } t = \frac{1}{4}\right)$.

[1] Point $(\overset{t}{0}, \overset{W}{1400})$

[2] Slope $= \frac{dW}{dt}\Big|_{(0,1400)} = \frac{1}{25}[1400 - 300]$

$\frac{dW}{dt} = \frac{1100}{25} = 44$

$W - 1400 = 44(t - 0)$ +1: tangent line

$W = 44(t - 0) + 1400$

$W(\frac{1}{4}) = 44(\frac{1}{4}) + 1400$

$W(\frac{1}{4}) = 1411$ +1: approximation

b.) Find $\frac{d^2W}{dt^2}$ in terms of W. Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.

$\frac{d}{dt}\left[\frac{dW}{dt}\right] = \left[\frac{1}{25}[W - 300]\right]\frac{d}{dt}$

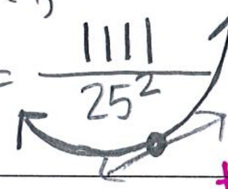
$\frac{d^2W}{dt^2} = \frac{1}{25}\left[(1)\frac{dW}{dt} - 0\right]$ +1: $\frac{d^2W}{dt^2}$

$\frac{d^2W}{dt^2} = \frac{1}{25}\left[\frac{1}{25}(W - 300)\right]$

$\frac{d^2W}{dt^2} = \frac{1}{25^2}[W - 300]$

$\frac{d^2W}{dt^2}\Big|_{(\frac{1}{4}, 1411)} = \frac{1}{25^2}[1411 - 300]$

$\frac{d^2W}{dt^2} = \frac{1111}{25^2}$



Underestimate because $W(t)$ is concave up.
+1: Conclusion w/ Reason

c.) Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ with the initial condition $W(\overset{t}{0}) = \overset{W}{1400}$.

$\frac{dW}{dt} = \frac{1}{25}[W - 300]$

$dW = \frac{1}{25}[W - 300]\,dt$

$\frac{1}{W - 300}\,dW = \frac{1}{25}\,dt$ +1: seperating variables

$\int \frac{1}{W - 300}\,dW = \int \frac{1}{25}\,dt$

$u = W - 300$

$du = dW$

$\int \frac{1}{u}\,du = \int \frac{1}{25}\,dt$

+1: antiderivative   +1: antiderivative

$\ln|W - 300| = \frac{1}{25}t + C$

$\ln|1400 - 300| = \frac{1}{25}(0) + C$ +1: uses initial condition

$C = \ln|1100|$

$\ln|W - 300| = \frac{1}{25}t + \ln 1100$

$e \qquad e$

$|W - 300| = 1100e^{\frac{1}{25}t}$

$W - 300 = \pm 1100e^{\frac{1}{25}t}$

$W = \pm 1100e^{\frac{1}{25}} + 300$

$1400 = \pm 1100e^{\frac{1}{25}(0)} + 300$

$W = 1100e^{\frac{1}{25}t} + 300$

+1: $y = f(x)$