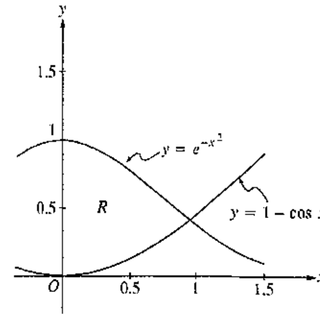


Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.

(a) Find the area of the region R.

(b) Find the volume of the solid generated when the region R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

Region R

$e^{-x^2} = 1 - \cos x$ at $x = 0.941944 = A$

1 : Correct limits in an integral in (a), (b), or (c).

(a) Area $= \int_0^A (e^{-x^2} - (1 - \cos x))\,dx$

$= 0.590$ or 0.591

2 { 1 : integrand; 1 : answer }

(b) Volume $= \pi \int_0^A \left(\left(e^{-x^2}\right)^2 - (1 - \cos x)^2 \right) dx$

$= 0.55596\pi = 1.746$ or 1.747

3 { 2 : integrand and constant; $< -1 >$ each error; 1 : answer }

(c) Volume $= \int_0^A \left(e^{-x^2} - (1 - \cos x)\right)^2 dx$

$= 0.461$

3 { 2 : integrand; $< -1 >$ each error; Note: 0/2 if not of the form $k \int_c^d (f(x) - g(x))^2\,dx$; 1 : answer }

1. A particle moves along the y–axis with velocity given by $v(t) = t\sin(t^2)$ for $t \geq 0$.
   (a) In which direction (up or down) is the particle moving at time $t = 1.5$? Why?
   (b) Find the acceleration of the particle at time $t = 1.5$. Is the velocity of the particle increasing at $t = 1.5$? Why or why not?
   (c) Given that $y(t)$ is the position of the particle at time t and that $y(0) = 3$, find $y(2)$.
   (d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

Solution	Scoring
(a) $v(1.5) = 1.5\sin(1.5^2) = 1.167$ Up, because $v(1.5) > 0$	1: answer and reason
(b) $a(t) = v'(t) = \sin t^2 + 2t^2\cos t^2$ $a(1.5) = v'(1.5) = -2.048$ or -2.049 No; v is decreasing at 1.5 because $v'(1.5) < 0$	2 { 1: $a(1.5)$ 1: conclusion and reason
(c) $y(t) = \int v(t)\,dt$ $= \int t\sin t^2\,dt = -\frac{\cos t^2}{2} + C$ $y(0) = 3 = -\frac{1}{2} + C \implies C = \frac{7}{2}$ $y(t) = -\frac{1}{2}\cos t^2 + \frac{7}{2}$ $y(2) = -\frac{1}{2}\cos 4 + \frac{7}{2} = 3.826$ or 3.827	3 { 1: $y(t) = \int v(t)\,dt$ 1: $y(t) = -\frac{1}{2}\cos t^2 + C$ 1: $y(2)$
(d) distance $= \int_0^2 \lvert v(t)\rvert\,dt = 1.173$ or $v(t) = t\sin t^2 = 0$ $t = 0$ or $t = \sqrt{\pi} \approx 1.772$ $y(0) = 3$; $y(\sqrt{\pi}) = 4$; $y(2) = 3.826$ or 3.827 $[y(\sqrt{\pi}) - y(0)] + [y(\sqrt{\pi}) - y(2)]$ $= 1.173$ or 1.174	3 { 1: limits of 0 and 2 on an integral of $v(t)$ or $\lvert v(t)\rvert$ or uses $y(0)$ and $y(2)$ to compute distance 1: handles change of direction at student's turning point 1: answer 0/1 if incorrect turning point

Question 4

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds, as shown in the table above.

(a) Find the average acceleration of rocket A over the time interval $0 \le t \le 80$ seconds. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_{10}^{70} v(t)\,dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)\,dt$.

(c) Rocket B is launched upward with an acceleration of $a(t) = \dfrac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.

Solution	Scoring
(a) Average acceleration of rocket A is $\dfrac{v(80)-v(0)}{80-0} = \dfrac{49-5}{80} = \dfrac{11}{20}\ \text{ft}/\text{sec}^2$	1 : answer
(b) Since the velocity is positive, $\int_{10}^{70} v(t)\,dt$ represents the distance, in feet, traveled by rocket A from $t = 10$ seconds to $t = 70$ seconds. A midpoint Riemann sum is $20[v(20)+v(40)+v(60)]$ $= 20[22+35+44] = 2020$ ft	3 : { 1 : explanation 1 : uses $v(20)$, $v(40)$, $v(60)$ 1 : value
(c) Let $v_B(t)$ be the velocity of rocket B at time t. $v_B(t) = \int \dfrac{3}{\sqrt{t+1}}\,dt = 6\sqrt{t+1} + C$ $2 = v_B(0) = 6 + C$ $v_B(t) = 6\sqrt{t+1} - 4$ $v_B(80) = 50 > 49 = v(80)$ Rocket B is traveling faster at time $t = 80$ seconds.	4 : { 1 : $6\sqrt{t+1}$ 1 : constant of integration 1 : uses initial condition 1 : finds $v_B(80)$, compares to $v(80)$, and draws a conclusion
Units of ft/sec^2 in (a) and ft in (b)	1 : units in (a) and (b)

Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

(a) How many gallons of water leak out of the tank from time $t = 0$ to $t = 3$ minutes?

(b) How many gallons of water are in the tank at time $t = 3$ minutes?

(c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t.

(d) At what time t, for $0 \le t \le 120$, is the amount of water in the tank a maximum? Justify your answer.

(a) Method 1: $\int_0^3 \sqrt{t+1}\,dt = \frac{2}{3}(t+1)^{3/2} \Big|_0^3 = \frac{14}{3}$

– or –

Method 2: $L(t)$ = gallons leaked in first t minutes

$\frac{dL}{dt} = \sqrt{t+1}; \quad L(t) = \frac{2}{3}(t+1)^{3/2} + C$

$L(0) = 0; \quad C = -\frac{2}{3}$

$L(t) = \frac{2}{3}(t+1)^{3/2} - \frac{2}{3}; \quad L(3) = \frac{14}{3}$

Scoring:

Method 1:

3 { 2 : definite integral (1 : limits; 1 : integrand); 1 : answer }

– or –

Method 2:

3 { 1 : antiderivative with C; 1 : solves for C using $L(0) = 0$; 1 : answer }

(b) $30 + 8 \cdot 3 - \frac{14}{3} = \frac{148}{3}$

Scoring: 1 : answer

(c) Method 1:

$$A(t) = 30 + \int_0^t \left(8 - \sqrt{x+1}\right)dx$$
$$= 30 + 8t - \int_0^t \sqrt{x+1}\,dx$$

– or –

Method 2:

$$\frac{dA}{dt} = 8 - \sqrt{t+1}$$
$$A(t) = 8t - \frac{2}{3}(t+1)^{3/2} + C$$
$$30 = 8(0) - \frac{2}{3}(0+1)^{3/2} + C; \quad C = \frac{92}{3}$$
$$A(t) = 8t - \frac{2}{3}(t+1)^{3/2} + \frac{92}{3}$$

Scoring:

Method 1:

2 { 1 : $30 + 8t$; 1 : $-\int_0^t \sqrt{x+1}\,dx$ }

– or –

Method 2:

2 { 1 : antiderivative with C; 1 : answer }

(d) $A'(t) = 8 - \sqrt{t+1} = 0$ when $t = 63$

$A'(t)$ is positive for $0 < t < 63$ and negative for $63 < t < 120$. Therefore there is a maximum at $t = 63$.

Scoring:

3 { 1 : sets $A'(t) = 0$; 1 : solves for t; 1 : justification }

Consider the differential equation $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$.

(a) Find a solution $y = f(x)$ to the differential equation satisfying $f(0) = \frac{1}{2}$.

(b) Find the domain and range of the function f found in part (a).

(a) $e^{2y}\,dy = 3x^2\,dx$

$$\frac{1}{2}e^{2y} = x^3 + C_1$$

$$e^{2y} = 2x^3 + C$$

$$y = \frac{1}{2}\ln\left(2x^3 + C\right)$$

$$\frac{1}{2} = \frac{1}{2}\ln(0 + C); \quad C = e$$

$$y = \frac{1}{2}\ln(2x^3 + e)$$

6 {
- 1 : separates variables
- 1 : antiderivative of dy term
- 1 : antiderivative of dx term
- 1 : constant of integration
- 1 : uses initial condition $f(0) = \frac{1}{2}$
- 1 : solves for y
- Note: 0/1 if y is not a logarithmic function of x

Note: max 3/6 [1-1-1-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

(b) Domain: $2x^3 + e > 0$

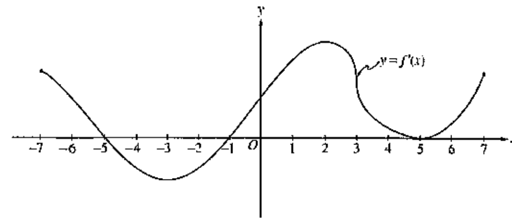
$$x^3 > -\frac{1}{2}e$$

$$x > \left(-\frac{1}{2}e\right)^{1/3} = -\left(\frac{1}{2}e\right)^{1/3}$$

Range: $-\infty < y < \infty$

3 {
- 1 : $2x^3 + e > 0$
- 1 : domain
  - Note: 0/1 if 0 is not in the domain
- 1 : range

Note: 0/3 if y is not a logarithmic function of x



The figure above shows the graph of f', the derivative of the function f, for $-7 \le x \le 7$. The graph of f' has horizontal tangent lines at $x = -3$, $x = 2$, and $x = 5$, and a vertical tangent line at $x = 3$.

(a) Find all values of x, for $-7 < x < 7$, at which f attains a relative minimum. Justify your answer.

(b) Find all values of x, for $-7 < x < 7$, at which f attains a relative maximum. Justify your answer.

(c) Find all values of x, for $-7 < x < 7$, at which $f''(x) < 0$.

(d) At what value of x, for $-7 \le x \le 7$, does f attain its absolute maximum? Justify your answer.

(a) $x = -1$

$f'(x)$ changes from negative to positive at $x = -1$

2 { 1 : answer; 1 : justification }

(b) $x = -5$

$f'(x)$ changes from positive to negative at $x = -5$

2 { 1 : answer; 1 : justification }

(c) $f''(x)$ exists and f' is decreasing on the intervals $(-7, -3)$, $(2, 3)$, and $(3, 5)$

2 { 1 : $(-7, -3)$; 1 : $(2, 3) \cup (3, 5)$ }

(d) $x = 7$

The absolute maximum must occur at $x = -5$ or at an endpoint.

$f(-5) > f(-7)$ because f is increasing on $(-7, -5)$

The graph of f' shows that the magnitude of the negative change in f from $x = -5$ to $x = -1$ is smaller than the positive change in f from $x = -1$ to $x = 7$. Therefore the net change in f is positive from $x = -5$ to $x = 7$, and $f(7) > f(-5)$. So $f(7)$ is the absolute maximum.

3 {
1 : answer
1 : identifies $x = -5$ and $x = 7$ as candidates
— or —
indicates that the graph of f increases, decreases, then increases
1 : justifies $f(7) > f(-5)$
}

AP Calculus
Calculator

Name________________
Date___________Period___

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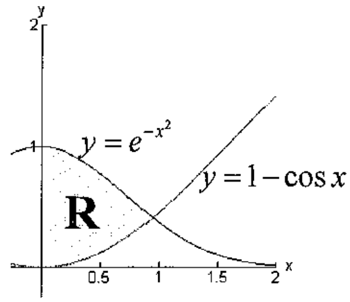

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Non-Calculator

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Non-Calculator

Name___________________
Date____________Period___

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AP Calculus
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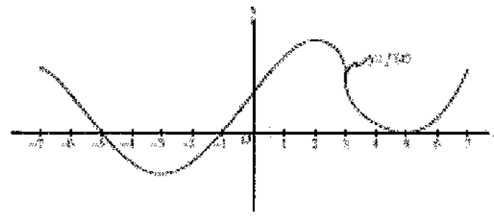

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