

Cartesian Coordinate System

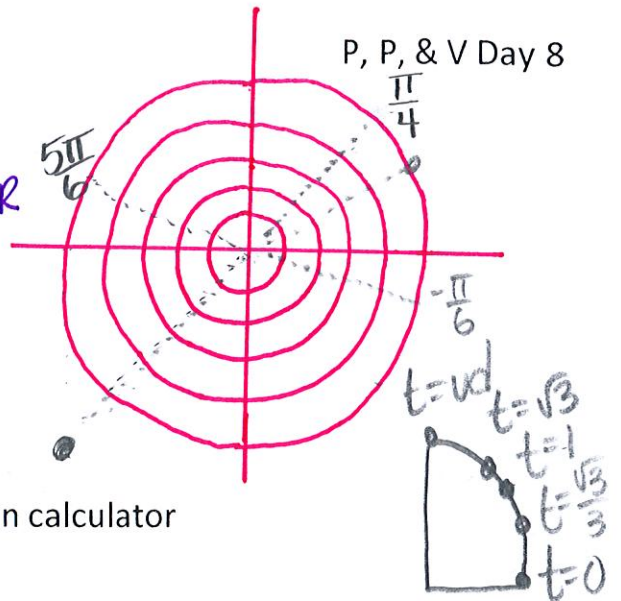
Rectangular \rightarrow Polar

$(x, y) \rightarrow (r, \theta)$

$r = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$\tan^{-1}(x) \rightarrow$ Your calculator will only give answer between $[-\frac{\pi}{2}, \frac{\pi}{2}]$
 $\sin^{-1}(x) \rightarrow$ Q1 & Q4
 $\cos^{-1}(x) \rightarrow$ Q1 & Q2



Example One: Convert each into polar coordinates

A.) $(-5, -5)$ by hand

B.) $(-\sqrt{3}, 1)$ in calculator

1. Identify Quadrant Q3

1. Q2

2. Use formulas

$R = \sqrt{(-5)^2 + (-5)^2}$
 $R = \sqrt{50} = 5\sqrt{2}$

2. $R = \sqrt{(-\sqrt{3})^2 + (1)^2} = \sqrt{4} = 2$

3. State ordered pair

$\theta = \tan^{-1}\left(\frac{-5}{-5}\right)$

$\theta = \tan^{-1}\left(\frac{1}{-\sqrt{3}}\right) = \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) = \frac{5\pi}{6}$

$(5\sqrt{2}, \frac{5\pi}{4})$

$\theta = \tan^{-1}(1)$

$(2, \frac{5\pi}{6})$

state 2 other ordered pairs

$\theta = \frac{5\pi}{4}$

$(2, \frac{17\pi}{6})$

$(5\sqrt{2}, \frac{13\pi}{4})$ $(-5\sqrt{2}, \frac{\pi}{4})$

$(-2, -\frac{\pi}{6})$

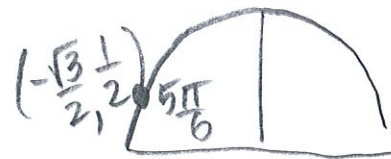
$(-2, \frac{11\pi}{6})$

Polar \rightarrow Rectangular

$(r, \theta) \rightarrow (x, y)$

$x = r \cos \theta$

$y = r \sin \theta$



Example Two: Convert each to rectangular

A.) $(-2, \frac{\pi}{4}) = (-\sqrt{2}, -\sqrt{2})$

B.) $(5, \frac{5\pi}{6}) = (-\frac{5\sqrt{3}}{2}, \frac{5}{2})$

$x = R \cos \theta = -2 \cos \frac{\pi}{4}$
 $= -2 \left(\frac{\sqrt{2}}{2}\right) = -\sqrt{2}$

$x = R \cos \theta = 5 \cos \frac{5\pi}{6} = 5 \left(-\frac{\sqrt{3}}{2}\right) = -\frac{5\sqrt{3}}{2}$

$y = R \sin \theta = 5 \sin \frac{5\pi}{6} = 5 \left(\frac{1}{2}\right) = \frac{5}{2}$

$y = R \sin \theta = -2 \sin \frac{\pi}{4}$
 $= -2 \left(\frac{\sqrt{2}}{2}\right) = -\sqrt{2}$

You use the same formulas when converting equations

$$r = \sqrt{x^2 + y^2}$$

$$x = r \cos \theta$$

$$r^2 = x^2 + y^2$$

$$y = r \sin \theta$$

Eqn Circle

$$(x-h)^2 + (y-k)^2 = R^2$$

Center: (h, k)

Radius: R

Example Three: Convert each to rectangular

A.) $(r)^2 = (4)^2$

$$r^2 = 16$$

$$x^2 + y^2 = 16$$

Circle: center $(0, 0)$
 $r = 4$

B.) $(r) = (6 \sin \theta) R$

$$R^2 = 6R \sin \theta$$

$$x^2 + y^2 = 6y$$

Complete the square

$$\left[\frac{1}{2} \text{middle}\right]^2$$

$$\left[\frac{1}{2}(-6)\right]^2$$

center $(0, 3)$
Radius = 3

$$x^2 + y^2 - 6y + 9 = 0 + 9 \Rightarrow x^2 + (y-3)^2 = 9$$

Example Four: Convert each to polar

A.) $x^2 + y^2 = 25$

$$R^2 = 25$$

$$\boxed{R = 5}$$

B.) $y = 3$

$$R \sin \theta = 3$$

$$R = \frac{3}{\sin \theta}$$

$$\boxed{R = 3 \csc \theta}$$

C.) $y^2 = x$

$$(R \sin \theta)^2 = R \cos \theta$$

$$\frac{R^2 \sin^2 \theta}{R \sin^2 \theta} = \frac{R \cos \theta}{R \sin^2 \theta}$$

$$R = \frac{\cos \theta}{\sin \theta \cdot \sin \theta}$$

$$\boxed{R = \cot \theta \csc \theta}$$

D.) $x = -2$

$$R \cos \theta = -2$$

$$R = \frac{-2}{\cos \theta}$$

$$\boxed{R = -2 \sec \theta}$$