x-component y-component

Suppose a particle's position is described by the vector $\vec{p}(t) = \langle 2t^2 - 8, \ln(2t-4) \rangle$. Answer the following.

1. State the domain of the vector function, $\vec{r}(t)$.

Look at each component: Anything excluded from the domain of each component is excluded from the domain of the vector.

$2t^2-8$ (Parabola) $d: (-\infty, \infty)$

$\ln(2t-4)$ domain $\ln(☺)$ ☺ > 0

$2t-4>0$

$d: t>2$

$\boxed{(2, \infty)}$

2. Find the vector that represents the velocity of the particle at any time t. take a derivative

position - velocity - acceleration

Integrate

$\frac{d}{dt}\left[p(t) = \langle 2t^2-8, \ln(2t-4) \rangle\right]$

$v(t) = \langle 4t, \frac{1}{2t-4}(2) \rangle$

$\boxed{v(t) = \langle 4t, \frac{1}{t-2} \rangle}$

3. How fast is the particle moving in the vertical direction when $t = 4$.

velocity y-component

$\frac{1}{4-2} = \boxed{\frac{1}{2}}$

vector = ⟨x-component, y-component⟩

left/Right movement OR horizontal

up/down movement OR vertical

4. Find the vector that represents the acceleration of the particle at any time t.

$\frac{d}{dt}\left[v(t) = \langle 4t, \frac{1}{t-2} \rangle = \langle 4t, 1(t-2)^{-1} \rangle\right]$

$a(t) = \langle 4, -1(t-2)^{-2}(1) \rangle$

$\boxed{a(t) = \langle 4, \frac{-1}{(t-2)^2} \rangle}$

5. Find a function $g(t)$ that represents the particle's speed at any time t.

Speed $= \|v(t)\| = \sqrt{[v(x)]^2 + [v(y)]^2} = \sqrt{(4t)^2 + \left(\frac{1}{t-2}\right)^2}$

Speed = magnitude of velocity

Speed $= \sqrt{16t^2 + \frac{1}{(t-2)^2}}$

$v(t)=\langle 4t, \frac{1}{t-2}\rangle$ $a(t)=\langle 4, \frac{-1}{(t-2)^2}\rangle$

Suppose a particle's position is described by the vector $\vec{r}(t)=\langle 2t^2-8, \ln(2t-4)\rangle$

6. Find the speed of the particle when $t = 4$.

$$\text{Speed}\Big|_{t=4} = \sqrt{16(4)^2+\frac{1}{(4-2)^2}} = \sqrt{256+\frac{1}{4}} = \boxed{16.0078}$$

dot product of $v(4)$ & $a(4)$ Dot Product: $x_1 \cdot x_2 + y_1 \cdot y_2$

7. Is the particle speeding up or slowing down when $t = 4$? DP: Is positive then speeding up

DP: Is negative then slowing down

$v(4) = \langle 16, \frac{1}{2}\rangle$

$a(4) = \langle 4, -\frac{1}{4}\rangle$ $DP = 16(4)+(\frac{1}{2})(-\frac{1}{4}) = 64 - \frac{1}{8} = \boxed{63.875 \text{ speeding up}}$

8. Find the total distance traveled by the particle on the interval $3 \le t \le 6$.

$$\text{Total distance} = \int_a^b \|v(t)\| = \int_a^b \sqrt{[v(x)]^2+[v(y)]^2}\,dt$$

$$\int_3^6 \sqrt{16t^2+\frac{1}{(t-2)^2}}\,dt = \boxed{54.025}$$

9. Find the total displacement by the particle on the interval $3 \le t \le 6$

Total displacement: Subtract position vectors & find magnitude.

$p(6) = \langle 2(36)-8, \ln(8)\rangle = \langle 64, \ln(8)\rangle$

$p(3) = \langle 2(9)-8, \ln(2)\rangle = \langle 10, \ln(2)\rangle$

$\langle 54, \ln 8 - \ln(2)\rangle$

$\langle 54, \ln(8/2)\rangle$

$\langle 54, \ln(4)\rangle$

displacement $= \sqrt{(54)^2+(\ln 4)^2}$

displacement $= \boxed{54.017}$

10. Sketch the path defined by $\vec{r}(t)$ on the interval $3 \le t \le 6$.

$p(3) = \langle 10, \ln 2\rangle = \langle 10, .7\rangle$

$p(4) = \langle 24, \ln 4\rangle = \langle 24, 1.4\rangle$

$p(5) = \langle 42, \ln 6\rangle = \langle 42, 1.8\rangle$

$p(6) = \langle 64, \ln 8\rangle = \langle 64, 2.1\rangle$