AP Calculus BC　　Name_______________

Taylor & Maclauring Estimating & Error Bound　　Day 2 Power

1-5: Compute $T_2(x)$ at x=a and use a calculator to compute the error $|f(x)-T_2(x)|$ at the given value of x.

1. $y=e^x$, $x=-0.5$, $a=0$ $T_2(x)=1+x+\frac{x^2}{2!}$

$T_2(-.5)=1+(-.5)+\frac{(-.5)^2}{2!}=.625$

$f(-.5)=e^{-.5}=.6065306597$

$\text{ERROR}=|.6065306597-.625|=\boxed{.0184693}$

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2. $y=\cos x$, $x=\frac{\pi}{12}$, $a=0$ $T_2(x)=1-\frac{x^2}{2!}$

$T_2(\frac{\pi}{12})=1-\frac{(\pi/12)^2}{2!}=.9657305403$

$f(\frac{\pi}{12})=\cos(\frac{\pi}{12})=.9659258263$

$\text{ERROR}=|.9659258263-.9657305403|=\boxed{.00019528}$

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3. $y=\ln x$, $x=.9$, $a=1$ $T_2(x)=(x-1)-\frac{1}{2}(x-1)^2$

$T_2(.9)=(-.1)-\frac{1}{2}(-.1)^2=-.105$

$f(.9)=\ln(.9)=-.1053605157$

$\text{ERROR}=|-.1053605157--.105|=\boxed{.000360515}$

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4. $y=\sin x$, $x=\frac{\pi}{20}$, $a=0$ $T_2(x)=x$

$T_2(\frac{\pi}{20})=\frac{\pi}{20}=.1570796327$

$f(\frac{\pi}{20})=\sin(\frac{\pi}{20})=.156434465$

$\text{ERROR}=|.156434465-.1570796327|=\boxed{.000645167}$

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5. $y=\frac{1}{1-x}$, $x=.25$, $a=0$ $T_2(x)=1+x+x^2$

$T_2(.25)=1+(.25)+(.25)^2=1.3125$

$f(.25)=\frac{1}{1-.25}=\frac{1}{.75}=\frac{4}{3}=1.\overline{3}$

$\text{ERROR}=|\frac{4}{3}-1.3125|=\boxed{.0208\overline{3}}$

$\text{Max ERROR} = \frac{K}{(n+1)!}|x-a|^{n+1}$ $K = f^{n+1}(u)$ $u = x$ or a

The bigger $f^{n+1}(x)$ or $f^{n+1}(a)$

6. Use the error bound to find the maximum possible size of $|\cos(0.3) - T_5(0.3)|$, where $T_5(x)$ is a Maclaurin polynomial. Verify your result with a calculator.

$f(x) = \cos x$
$f'(x) = -\sin x$
$f''(x) = -\cos x$
$f^3(x) = \sin x$
$f^4(x) = \cos x$
$f^5(x) = -\sin x$
$f^6(x) = -\cos x$

$f^6(0) = |-\cos(0)| = 1$
$f^6(.3) = |-\cos(.3)| = .95533$
$K = 1$ (larger of the two)

$\text{Max ERROR} = \frac{1}{6!}|.3-0|^6 = \boxed{.0000010125}$

7. Use the error bound to find the maximum possible size of $|\sin(0.25) - T_6(0.25)|$, where $T_6(x)$ is a Maclaurin polynomial. Verify your result with a calculator.

$f(x) = \sin x$
$f'(x) = \cos x$
$f^2(x) = -\sin x$
$f^3(x) = -\cos x$
$f^4(x) = \sin x$
$f^5(x) = \cos x$
$f^6(x) = -\sin x$
$f^7(x) = -\cos x$

$f^7(0) = |-\cos(0)| = 1$
$f^7(.25) = |-\cos(.25)| = .9689124$
$K = 1$ (larger of the 2)

$\text{Max ERROR} = \frac{1}{7!}|.25-0|^7 = \boxed{.000000012110}$

8. Use the error bound to find the maximum possible size of $|e^{.1} - T_3(0.1)|$, where $T_3(x)$ is a Maclaurin polynomial. Verify your result with a calculator.

$f(x) = e^x$
$f'(x) = e^x$
$f^2(x) = e^x$
$f^3(x) = e^x$
$f^4(x) = e^x$

$f^4(0) = |e^0| = 1$
$f^4(.1) = |e^{.1}| = 1.105170918$
$K = 1.105170918$ (larger of the 2)

$\text{Max ERROR} = \frac{1.1051709}{4!}|.1-0|^4 = \boxed{.0000046048788}$

9. Use the error bound to find the maximum possible size of $\left|\frac{1}{1-(.25)} - T_2(0.25)\right|$, where $T_2(x)$ is a Maclaurin polynomial. Verify your result with a calculator.

$f(x) = \frac{1}{1-x} = (1-x)^{-1}$
$f'(x) = -1(1-x)^{-2}(-1) = 1(1-x)^{-2}$
$f''(x) = -2(1-x)^{-3}(-1) = 2(1-x)^{-3}$
$f^3(x) = -6(1-x)^{-4}(-1) = 6(1-x)^{-4}$

$f^3(0) = \left|\frac{6}{(1-0)^4}\right| = 6$
$f^3(.25) = \left|\frac{6}{(1-.25)^4}\right| = 18.96296296$
K (larger of the 2)

$\text{Max ERROR} = \frac{18.96296296}{3!}|.25-0|^3 = \boxed{.049382716}$

10. Use the error bound to find the maximum possible size of $|\ln(1.4) - T_4(1.4)|$, where $T_4(x)$ is a Taylor polynomial centered at a=1. Verify your result with a calculator.

$f(x) = \ln x$
$f'(x) = \frac{1}{x} = x^{-1}$
$f''(x) = -1x^{-2}$
$f^3(x) = 2x^{-3}$
$f^4(x) = -6x^{-4}$
$f^5(x) = 24x^{-5}$

$f^5(1) = \left|\frac{24}{(1)^5}\right| = 24$
$f^5(1.4) = \left|\frac{24}{(1.4)^5}\right| = 4.46242637$
$K = 24$ (larger of the 2)

$\text{Max ERROR} = \frac{24}{5!}|1.4-1|^5 = .2(.4)^5 = \boxed{.002048}$