AP Calculus BC Name________________

Homework: Taylor Polynomials Power Day 6

1-6: Calculate the Taylor polynomial $T_2(x)$ and $T_3(x)$ centered at $x = a$ for the given function and the given value of a.

Final Answers only

1. $f(x) = sinx,\ a = 0$

$T_2(x) = x.$

$T_3(x) = x - \frac{x^3}{3!}$

2. $f(x) = sinx,\ a = \frac{\pi}{2}$

$T_2(x) = 1 - \frac{(x-\frac{\pi}{2})^2}{2}$

$T_3(x) = 1 - \frac{(x-\frac{\pi}{2})^2}{2}$

3. $f(x) = \frac{1}{1+x^2},\ a = 0$

$T_2(x) = 1 - x^2$

$T_3(x) = 1 - x^2$

4. $f(x) = e^x,\ a = ln2$

$T_2(x) = 2 + 2(x - \ln 2) + (x - \ln 2)^2$

$T_3(x) = 2 + 2(x - \ln 2) + (x - \ln 2)^2 + \frac{(x - \ln 2)^3}{3}$

5. $f(x) = x^2 e^{-x}$, $a = 1$

$$T_2(x) = \frac{1}{e} + \frac{1}{e}(x-1) - \frac{1}{e \cdot 2!}(x-1)^2$$

$$T_3(x) = \frac{1}{e} + \frac{1}{e}(x-1) + \frac{1}{e \cdot 2!}(x-1)^2 - \frac{1}{e \cdot 3!}(x-1)^3$$

6. $f(x) = \ln(x + 1)$, , $a = 0$

$$T_2(x) = x - \frac{1}{2}x^2$$

$$T_3(x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3$$

7. 1995 BC4

Let f be a function that has derivatives of all orders for all real numbers. Assume $f(1) = 3$, $f'(1) = -2$, $f''(1) = 2$, and $f'''(1) = 4$

a. Write the second degree Taylor polynomial for f about $x = 1$ and use it to approximate $f(0.7)$.

$$T_{(2)}(x) = 3 - 2(x-1) + \frac{2(x-1)^2}{2} = 3 - 2(x-1) + (x-1)^2$$

$$T_2(.7) = 3 - 2(.7-1) + (.7-1)^2 = 3.69$$

b. Write the third-degree Taylor polynomial for f about $x = 1$ and use it to approximate $f(1.2)$.

$$T_3(x) = 3 - 2(x-1) + (x-1)^2 + \frac{2(x-1)^3}{3}$$

$$T_3(1.2) = 3 - 2(1.2-1) + (1.2-1)^2 + \frac{2(1.2-1)^3}{3} = 2.6453$$

c. Write the second-degree Taylor polynomial for f', the derivative of f, about $x = 1$ and use it to approximate $f'(1.2)$.

$$f(x) = 3 - 2(x-1) + (x-1)^2 + \frac{2(x-1)^3}{3} \ldots$$ take derivative

$$f'(x) = -2 + 2(x-1) + 2(x-1)^2$$

$$f'(1.2) = -2 + 2(1.2-1) + 2(1.2-1)^2 = -1.52$$