Notes: Taylor Series Power Series Day 5

Warm-up: Find the formula for each:

A. 2, 4, 6, 8, 10 .....

B. 14, 16, 18, 20, 22, ...

C. 10, 13, 16, 19, 22, ...

Taylor Series: centered at $x = c$

$$\sum_{n=0}^{\infty}\frac{f^{(n)}(c)}{n!}(x-c)^n = \frac{f(c)}{0!}+\frac{f'(c)}{1!}(x-c)+\frac{f''(c)}{2!}(x-c)^2+\frac{f'''(c)}{3!}(x-c)^3+\cdots\ldots\ldots\frac{f^n(c)}{n!}(x-c)^n$$

Example One: Find the first five terms and the general term of the Taylor Series for $f(x) = \cos x$ centered at $c = \pi$.

$f(x) = \cos x \quad f(\pi) = -1$

$f'(x) = -\sin x \quad f'(\pi) = 0$

$f''(x) = -\cos x \quad f''(\pi) = 1$

$f^3(x) = \sin x \quad f^3(\pi) = 0$

$f^4(x) = \cos x \quad f^4(\pi) = -1$

$\frac{-1}{0!}(x-\pi)^0 + \frac{0}{1!}\cancel{(x-\pi)^1}_{0} + \frac{1}{2!}(x-\pi)^2 + \frac{0}{3!}\cancel{(x-\pi)^3}_{0} + \frac{-1}{4!}(x-\pi)^4 + \ldots$

$T(x) = -1 + \frac{(x-\pi)^2}{2!} - \frac{(x-\pi)^4}{4!} + \frac{(x-\pi)^6}{6!} - \frac{(x-\pi)^8}{8!} + \ldots$

$n=0 \quad n=1 \quad n=2 \quad n=3 \quad n=4$

$$= \sum_{n=0}^{\infty}\frac{(-1)^{n+1}(x-\pi)^{2n}}{(2n)!}$$

Example Two: Find the first five terms and the general term of the Taylor Series for $f(x) = \frac{1}{x}$ centered at c = 1.

$T(x) = 1 - (x-1)^1 + (x-1)^2 - (x-1)^3 + (x-1)^4 + \ldots$

$n=0 \quad n=1 \quad n=2 \quad n=3 \quad n=4$

$f(x) = \frac{1}{x} = x^{-1} \quad f(1) = \frac{1}{1} = 1$

$f'(x) = -1x^{-2} \quad f'(1) = \frac{-1}{(1)^2} = -1$

$f^2(x) = 2x^{-3} \quad f^2(1) = \frac{2}{(1)^3} = 2$

$f^3(x) = -6x^{-4} \quad f^3(1) = \frac{-6}{(1)^4} = -6$

$f^4(x) = 24x^{-5} \quad f^4(1) = \frac{24}{(1)^5} = 24$

$$= \sum_{n=0}^{\infty}(-1)^n(x-1)^n$$

$T(x) = \frac{1}{\cancel{0!}}(x-1)^0 + \frac{-1}{\cancel{1!}}(x-1)^1 + \frac{\cancel{2}}{\cancel{2!}}(x-1)^2 + \frac{-\cancel{6}}{\cancel{3!}}(x-1)^3 + \frac{\cancel{24}}{\cancel{4!}}(x-1)^4 + \ldots$

$$f(x)=e^{3(x+1)-3}=e^{-3}\cdot e^{3(x+1)}=\sum_{n=0}^{\infty}\frac{(3(x+1))^n}{n!}\cdot e^{-3}=\sum_{n=0}^{\infty}\frac{3^n e^{-3}(x+1)^n}{n!}$$

Example Three: Find the first five terms and the general term of the Taylor Series for $f(x)=e^{3x}$ centered at c = -1.

$$T(x)=\frac{e^{-3}}{0!}(x+1)^0+\frac{3e^{-3}}{1!}(x+1)^1+\frac{3^2e^{-3}}{2!}(x+1)^2+\frac{3^3e^{-3}}{3!}(x+1)^3+\frac{3^4e^{-3}}{4!}(x+1)^4$$

n=0 n=1 n=2

$f(x)=e^{3x}$ $f(-1)=e^{-3}$

$f'(x)=3e^{3x}$ $f'(-1)=3e^{-3}$

$f^2(x)=3^2e^{3x}$ $f^2(-1)=3^2e^{-3}$

$f^3(x)=3^3e^{3x}$ $f^3(-1)=3^3e^{-3}$

$f^4(x)=3^4e^{3x}$ $f^4(-1)=3^4e^{-3}$

$$\sum_{n=0}^{\infty}\frac{3^n e^{-3}(x+1)^n}{n!}$$

Example Four: Find the first five terms and the general term of the Taylor Series for

$f(x)=\frac{1}{1+x}$ $=1(1+x)^{-1}$ centered at c = 2. $f(2)=\frac{1}{3}$

$f'(x)=-1(1+x)^{-2}(1)$ $f'(2)=\frac{-1}{3^2}$

$f^2(x)=2(1+x)^{-3}(1)$ $f''(2)=\frac{2}{3^3}$

$f^3(x)=-6(1+x)^{-4}$ $f^3(2)=\frac{-6}{3^4}$

$f^4(x)=24(1+x)^{-5}$ $f^4(2)=\frac{24}{3^5}$

$$T(x)=\frac{1}{3\cdot 0!}(x-2)^0-\frac{1}{3^2\cdot\cancel{1!}}(x-2)^1+\frac{\cancel{2}}{3^3\cdot\cancel{2!}}(x-2)^2+\frac{-\cancel{6}}{3^4\cdot\cancel{3!}}(x-2)^3+\frac{\cancel{24}}{3^5\cdot\cancel{4!}}(x-2)^4+\ldots$$

$$T(x)=\frac{1}{3}-\frac{(x-2)}{3^2}+\frac{(x-2)^2}{3^3}-\frac{(x-2)^3}{3^4}+\frac{(x-2)^4}{3^5}+\ldots$$

n=0 n=1 n=2 n=3 n=4

$$\sum_{n=0}^{\infty}\frac{(-1)^n(x-2)^n}{3^{n+1}}$$

$$f(x)=\frac{1}{1+x}=\frac{1}{3+(x-2)}$$

$$=\frac{1}{3\left[1+\frac{x-2}{3}\right]}=\sum_{n=0}^{\infty}(-1)^n\left(\frac{x-2}{3}\right)^n\frac{1}{3}$$

$$=\sum_{n=0}^{\infty}\frac{(-1)^n(x-2)^n}{3^{n+1}}$$