AP Calculus BC

Series from known Maclaurin Series

Name____________________

Power Series Day 4

Find a power series representation for the function and determine the interval of convergence.

1. $f(x) = \dfrac{1}{1+x}$

$= 1 - x + x^2 - x^3 + x^4 - \ldots = \sum_{n=0}^{\infty} (-1)^n x^n$

IOC: $(-1, 1)$

2. $f(x) = \dfrac{5}{1-4x^2}$

$= 5 + 20x^2 + 80x^4 + 320x^6 + \ldots = \sum_{n=0}^{\infty} 5(4x^2)^n$

$= \sum_{n=0}^{\infty} 5 \cdot 4^n x^{2n}$

IOC: $\left(-\frac{1}{2}, \frac{1}{2}\right)$

$|4x^2| < 1$

3. $f(x) = \dfrac{2}{3-x}$

$= \frac{2}{3} + \frac{2x}{9} + \frac{2x^2}{27} + \frac{2x^3}{81} + \ldots = \sum_{n=0}^{\infty} \frac{2x^n}{3^{n+1}}$

IOC: $(-3, 3)$

4. $f(x) = \dfrac{x}{9+x^2}$

$= \frac{x}{9} - \frac{x^3}{81} + \frac{x^5}{729} - \frac{x^7}{6561} + \ldots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{9^{n+1}}$

IOC: $(-3, 3)$

5. $f(x) = \dfrac{x}{2x^2+1}$

$= x - 2x^3 + 4x^5 - 8x^7 + \ldots = \sum_{n=0}^{\infty} (-1)^n 2^n x^{2n+1}$

IOC:

$\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

6. $f(x) = \dfrac{1}{x+10}$

$= \frac{1}{10} - \frac{x}{100} + \frac{x^2}{1000} - \frac{x^3}{10000} + \ldots = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{10^{n+1}}$

IOC: $(-10, 10)$

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Power Series Day 4

Use a Maclaurin series to obtain the Maclaurin series for the given function.

7. $f(x) = \sin \pi x$

$$= \pi x - \frac{(\pi x)^3}{3!} + \frac{(\pi x)^5}{5!} - \frac{(\pi x)^7}{7!} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1} x^{2n+1}}{(2n+1)!}$$

8. $f(x) = x\cos\left(\frac{1}{2}x^2\right)$

$$= x - \frac{x^5}{4 \cdot 2!} + \frac{x^9}{16 \cdot 4!} - \frac{x^{13}}{64 \cdot 6!} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+1}}{4^n (2n)!}$$

9. $f(x) = \ln(1 - x^2)$

$$= -x^2 - \frac{x^4}{2} - \frac{x^6}{3} - \frac{x^8}{4} - \dots = \sum_{n=0}^{\infty} \frac{-x^{2n+2}}{n+1}$$

10. $f(x) = e^{4x}$

$$= 1 + 4x + \frac{16x^2}{2!} + \frac{64x^3}{3!} + \dots = \sum_{n=0}^{\infty} \frac{(4x)^n}{n!} = \sum_{n=0}^{\infty} \frac{4^n x^n}{n!}$$

11. $f(x) = x^2 e^{x^2}$

$$= x^2 + x^4 + \frac{x^6}{2!} + \frac{x^8}{3!} + \dots = \sum_{n=0}^{\infty} \frac{x^{2n+2}}{n!}$$

12. $f(x) = \tan^{-1} x^2$

$$= x^2 - \frac{x^6}{3} + \frac{x^{10}}{5} - \frac{x^{14}}{7} + \dots = \sum_{n=0}^{\infty} \frac{x^{4n+2}}{2n+1}$$

13. $f(x) = e^x - \cos x$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$e^x - \cos x = x + x^2 + \frac{x^3}{3!} + \frac{x^5}{120} + \frac{2x^6}{6!} + \dots = \sum_{n=0}^{\infty} \frac{x^n}{n!} + \frac{(-1)^{n+1} x^{2n}}{(2n)!}$$