AP Calculus BC
Series from known Maclaurin Series

Name Homework Guide
Power Series Day 4

Find a power series representation for the function and determine the interval of convergence.

1. $f(x)=\frac{1}{1+x}$

Know $\frac{1}{1+x}=\sum_{n=0}^{\infty}(-1)^n x^n$

I.O.C $=(-1,1)$

2. $f(x)=\frac{5}{1-4x^2}$

3. $f(x)=\frac{2}{3-x}$

Know $\frac{1}{1-x}=\sum_{n=0}^{\infty}(x)^n$

$f(x)=\frac{2}{3}\left[\frac{1}{1-\frac{x}{3}}\right]=\sum_{n=0}^{\infty}\frac{2}{3}\cdot\left(\frac{x}{3}\right)^n=\sum_{n=0}^{\infty}\frac{2x^n}{3^{n+1}}$

I.O.C $\left|\frac{x}{3}\right|<1$

$\frac{x}{3}<1$ & $\frac{x}{3}>-1$

$x<3$ $x>-3$

$(-3,3)$

4. $f(x)=\frac{x}{9+x^2}$

5. $f(x)=\frac{x}{2x^2+1}$

Know: $\frac{1}{1+x}=\sum_{n=0}^{\infty}(-1)^n x^n$

$f(x)=\frac{x}{1+2x^2}=x\left[\frac{1}{1+2x^2}\right]=\sum_{n=0}^{\infty}(-1)^n\cdot x\cdot(2x^2)^n$

$=\sum_{n=0}^{\infty}(-1)^n 2^n x^{2n+1}$

I.O.C $|2x^2|<1$

$2x^2<1$ & $2x^2>-1$

$x^2<1/2$ $\cancel{x^2>-1/2}$

$x<\sqrt{1/2}$ & $x>-\sqrt{1/2}$ garbage

$\left(\frac{-1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$

6. $f(x)=\frac{1}{x+10}$

AP Calculus BC
Series from known Maclaurin Series

Name________________________
Power Series Day 4

7-13: Use a Maclaurin series to obtain the Maclaurin series for the given function.

7. $f(x) = \sin \pi x$ Know: $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

$\sin(\pi x) = \sum_{n=0}^{\infty} \frac{(-1)^n (\pi x)^{2n+1}}{(2n+1)!}$

$= \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1} x^{2n+1}}{(2n+1)!}$

I.O.C $= (-\infty, \infty)$

$R = \lim_{n\to\infty} \left| \frac{\pi^{2n+3} x^{2n+3}}{(2n+3)!} \cdot \frac{(2n+1)!}{\pi^{2n+1} x^{2n+1}} \right|$

$= \lim_{n\to\infty} \left| \frac{\cancel{\pi^{2n}} \pi^{\cancel{3}^2} \cancel{x^{2n}} x^{\cancel{3}^2} \cancel{(2n+1)!}}{\cancel{\pi^{2n}} \cdot \cancel{\pi^1} \cdot \cancel{x^{2n}} \cdot \cancel{x^1} (2n+3)(2n+2)\cancel{(2n+1)!}} \right| = |x^2| \lim_{n\to\infty} \left| \frac{\pi^2}{(2n+3)(2n+2)} \right| = x^2(0) < 1$

8. $f(x) = x \cos\left(\frac{1}{2}x^2\right)$

9. $f(x) = \ln(1-x^2)$ Know: $\ln(1+x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}$

$f(x) = \ln(1+(-x^2))$

$\sum_{n=0}^{\infty} \frac{(-1)^n(-x^2)^{n+1}}{(n+1)} = \sum_{n=0}^{\infty} \frac{\cancel{(-1)^n} \cancel{(-1)^n} (-1)^1 (x^2)^{n+1}}{n+1} = \sum_{n=0}^{\infty} \frac{-x^{2n+2}}{n+1}$

10. $f(x) = e^{4x}$

AP Calculus BC

Series from known Maclaurin Series

Name________________________

Power Series Day 4

11. $f(x) = x^2 e^{x^2}$ Know: $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$$f(x) = x^2 e^{x^2} = \sum_{n=0}^{\infty} \frac{x^2 \cdot (x^2)^n}{n!} = \sum_{n=0}^{\infty} \frac{x^{2n+2}}{n!}$$

12. $f(x) = \tan^{-1} x^2$

13. $f(x) = e^x - \cos x$ Know: $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ & $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} - \frac{(-1)^n x^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{x^n}{n!} + \frac{(-1)^{n+1} x^{2n}}{(2n)!}$$

Review:

R1. Consider the series $\sum_{n=1}^{\infty} \frac{e^n}{n!}$. If the ratio test is applied to the series, which of the following inequalities results, implying the series converges?

a. $\lim_{n\to\infty} \frac{e}{n!} < 1$

b. $\lim_{n\to\infty} \frac{n!}{e} < 1$

c. $\lim_{n\to\infty} \frac{n+1}{e} < 1$

d. $\lim_{n\to\infty} \frac{e}{n+1} < 1$

e. $\lim_{n\to\infty} \frac{e}{(n+1)!} < 1$

$$R = \lim_{n\to\infty} \left| \frac{e^{n+1}}{(n+1)!} \cdot \frac{n!}{e^n} \right|$$

$$R = \lim_{n\to\infty} \left| \frac{\cancel{e^n} e^1 \cdot \cancel{n!}}{\cancel{e^n}(n+1)\cdot \cancel{n!}} \right|$$

$$R = \lim_{n\to\infty} \frac{e}{n+1} < 1$$

R2. The interval of convergence of the power series $\sum_{n=0}^{\infty} \left(\frac{x}{3}\right)^n$ is

a. $[0]$

b. $\left(-\frac{1}{3}, \frac{1}{3}\right)$

c. $[-3,3]$

d. $(-3,3)$

e. $(-\infty,\infty)$

$$\left|\frac{x}{3}\right| < 1$$

$$\frac{x}{3} < 1 \ \& \ \frac{x}{3} > -1$$

$$x < 3 \ \& \ x > -3$$

$$(-3,3)$$

AP Calculus BC

Series from known Maclaurin Series

Name________________________

Power Series Day 4

Review Continued

R3. The sum of the geometric series $1+\frac{2}{5}+\frac{4}{25}+\frac{8}{125}+\frac{16}{625}+\ldots$ is $R=\frac{2}{5}$

a. $\frac{3}{5}$

b. $\frac{2}{3}$

c. $\frac{5}{3}$

d. $\frac{3}{2}$

e. $\frac{5}{2}$

Sum = $\frac{\text{first term}}{1-\text{ratio}}$

Sum = $\frac{1}{1-2/5}$

$= \frac{1}{3/5}$

$= 5/3$

R4. Which of the following sequences converge?

$I.\left\{\frac{5n}{2n-1}\right\}$ $II.\left\{\frac{e^n}{n}\right\}$ $III.\left\{\frac{e^n}{1+e^n}\right\}$

~~a. I only~~

b. II only

~~c. I & II only~~

~~d. I & III only~~

~~e. I, II, & III~~

$\lim_{n\to\infty} \frac{5n}{2n-1} = \frac{5}{2} \neq 0$

$\therefore$ diverges

Answers:

1. $\sum_{n=0}^{\infty}(-1)^n x^n$ $I.O.C(-1,1)$

2. $\sum_{n=0}^{\infty}5\cdot 4^n x^{2n}$ $I.O.C\left(-\frac{1}{2},\frac{1}{2}\right)$

3. $\sum_{n=0}^{\infty}\frac{2x^n}{3^{n+1}}$ $I.O.C(-3,3)$

4. $\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{9^{n+1}}$ $I.O.C(-3,3)$

5. $\sum_{n=0}^{\infty}(-1)^n 2^n x^{2n+1}$ $I.O.C\left(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$

6. $\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{10^{n+1}}$ $I.O.C(-10,10)$

7. $\sum_{n=0}^{\infty}\frac{(-1)^n \pi^{2n+1}x^{2n+1}}{(2n+1)!}$

8. $\sum_{n=0}^{\infty}\frac{(-1)^n x^{4n+1}}{4^n(2n)!}$

9. $\sum_{n=0}^{\infty}\frac{-x^{2n+2}}{n+1}$

10. $\sum_{n=0}^{\infty}\frac{4^n x^n}{n!}$

11. $\sum_{n=0}^{\infty}\frac{x^{2n+2}}{n!}$

12. $\sum_{n=0}^{\infty}\frac{x^{4n+2}}{2n+1}$

13. $\sum_{n=0}^{\infty}\frac{x^n}{n!}+\frac{(-1)^{n+1}x^{2n}}{(2n)!}$

R1. D R2. D R3. C R4. B