

Find a power series representation for the function and determine the interval of convergence.

1. $f(x) = \frac{1}{1+x}$

2. $f(x) = \frac{5}{1-4x^2}$

3. $f(x) = \frac{2}{3-x}$

4. $f(x) = \frac{x}{9+x^2}$

5. $f(x) = \frac{x}{2x^2+1}$

6. $f(x) = \frac{1}{x+10}$

7-13: Use a Maclaurin series to obtain the Maclaurin series for the given function.

7. $f(x) = \sin \pi x$

8. $f(x) = x \cos\left(\frac{1}{2}x^2\right)$

9. $f(x) = \ln(1-x^2)$

10. $f(x) = e^{4x}$

11. $f(x) = x^2 e^{x^2}$

12. $f(x) = \tan^{-1} x^2$

13. $f(x) = e^x - \cos x$

Review:

R1. Consider the series $\sum_{n=1}^{\infty} \frac{e^n}{n!}$. If the ratio test is applied to the series, which of the following inequalities results, implying the series converges?

- a. $\lim_{n \rightarrow \infty} \frac{e}{n!} < 1$
- b. $\lim_{n \rightarrow \infty} \frac{n!}{e} < 1$
- c. $\lim_{n \rightarrow \infty} \frac{n+1}{e} < 1$
- d. $\lim_{n \rightarrow \infty} \frac{e}{n+1} < 1$
- e. $\lim_{n \rightarrow \infty} \frac{e}{(n+1)!} < 1$

R2. The interval of convergence of the power series $\sum_{n=0}^{\infty} \left(\frac{x}{3}\right)^n$ is

- a. $[0]$
- b. $\left(-\frac{1}{3}, \frac{1}{3}\right)$
- c. $(-3, 3)$
- d. $[-3, 3]$
- e. $(-\infty, \infty)$

Review Continued

R3. The sum of the geometric series

$$1 + \frac{2}{5} + \frac{4}{25} + \frac{8}{125} + \frac{16}{625} + \dots \text{ is}$$

- a. $\frac{3}{5}$
 b. $\frac{2}{3}$
 c. $\frac{5}{3}$
 d. $\frac{3}{2}$
 e. $\frac{5}{2}$

R4. Which of the following sequences converge?

$$I. \left\{ \frac{5n}{2n-1} \right\} \quad II. \left\{ \frac{e^n}{n} \right\} \quad III. \left\{ \frac{e^n}{1+e^n} \right\}$$

- a. I only
 b. II only
 c. I & II only
 d. I & III only
 e. I, II, & III

Answers:

1. $\sum_{n=0}^{\infty} (-1)^n x^n \quad I.O.C(-1,1)$

3. $\sum_{n=0}^{\infty} \frac{2x^n}{3^{n+1}} \quad I.O.C(-3,3)$

5. $\sum_{n=0}^{\infty} (-1)^n 2^n x^{2n+1} \quad I.O.C\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

7. $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1} x^{2n+1}}{(2n+1)!}$

9. $\sum_{n=0}^{\infty} \frac{-x^{2n+2}}{n+1}$

11. $\sum_{n=0}^{\infty} \frac{x^{2n+2}}{n!}$

13. $\sum_{n=0}^{\infty} \frac{x^n}{n!} + \frac{(-1)^{n+1} x^{2n}}{(2n)!}$

R1. D

R2. D

2. $\sum_{n=0}^{\infty} 5 \cdot 4^n x^{2n} \quad I.O.C\left(-\frac{1}{2}, \frac{1}{2}\right)$

4. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{9^{n+1}} \quad I.O.C(-3,3)$

6. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{10^{n+1}} \quad I.O.C(-10,10)$

8. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+1}}{4^n (2n)!}$

10. $\sum_{n=0}^{\infty} \frac{4^n x^n}{n!}$

12. $\sum_{n=0}^{\infty} \frac{x^{4n+2}}{2n+1}$

R3. C

R4. B