AP Calculus BC

Maclaurin Series

Name______________________________

Power Series Day 3

MEMORIZE THESE MACLAURIN SERIES

f(x)	Maclaurin Series	General Term
e^x	$= 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\cdots\frac{x^n}{n!}$	$\sum_{n=0}^{\infty}\frac{x^n}{n!}$
$\sin x$	$= x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots\frac{(-1)^n x^{2n+1}}{(2n+1)!}$	$\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)!}$
$\cos x$	$= 1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots\frac{(-1)^n x^{2n}}{(2n)!}$	$\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}$
$\ln(1+x)$	$= x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots\frac{(-1)^n x^{n+1}}{n+1}$	$\sum_{n=0}^{\infty}\frac{(-1)^n x^{n+1}}{n+1}$
$\frac{1}{1-x}$	$= 1+x+x^2+x^3+\cdots+x^n$	$\sum_{n=0}^{\infty}x^n$
$\frac{1}{1+x}$	$= 1-x+x^2-x^3+x^4-\cdots(-1)^n x^n$	$\sum_{n=0}^{\infty}(-1)^n x^n$
$\tan^{-1}x$	$= x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\cdots\frac{(-1)^n x^{2n+1}}{2n+1}$	$\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{2n+1}$

1. Write out the first four terms of the Maclaurin of f(x) if

$f(0) = 2, \quad f'(0) = 3, \quad f''(0) = 4, \quad f'''(0) = 12$

$$= 2+3(x-0)+\frac{4(x-0)^2}{2!}+\frac{12(x-0)^3}{3!}$$

$$= 2+3x+2x^2+2x^3$$

2-5: Find the Maclaurin series for f(x) using the definition of a Maclaurin series. [Assume that f has a power series expansion.] Also find the associated radius of convergence.

2. $f(x)=(1-x)^{-2}$

$f(x)=(1-x)^{-2}$ $\quad f(0)=1$

$f'(x)=-2(1-x)^{-3}(-1)$ $\quad f'(0)=2$

$=2(1-x)^{-3}$

$f''(x)=-6(1-x)^{-4}(-1)$ $\quad f''(0)=6$

$=6(1-x)^{-4}$

$f'''(x)=-24(1-x)^{-5}(-1)$ $\quad f'''(0)=24$

$=24(1-x)^{-5}$

$f^{(4)}(x)=-120(1-x)^{-6}(-1)$ $\quad f^{(4)}(0)=120$

$=120(1-x)^{-6}$

$$= 1+2x+\frac{6x^2}{2!}+\frac{24x^3}{3!}+\frac{120x^4}{4!}+\cdots$$

$$= 1+2x+3x^2+4x^3+5x^4+\cdots\sum_{n=0}^{\infty}nx^{n-1}$$

$(-1,1)$

3. $f(x) = \ln(x)$

$f(x) = \ln x \qquad f(0) = \emptyset$

$f'(x) = \frac{1}{x} = x^{-1} \qquad \emptyset$

$f''(x) = -x^{-2} \qquad \emptyset$

$f'''(x) = 2x^{-3} \qquad \emptyset$

hmm..

Why does this not make sense to center at 0 ?

asymptote

So... we can use $\ln(1+x)$

$\ln x = \ln(1+(x-1))$

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots = \sum \frac{(-1)^n x^{n+1}}{n+1}$

$\therefore \ln(1+(x-1)) = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \dots$

$= \sum_{n=0}^{\infty} \frac{(-1)^n (x-1)^n}{n+1}$

$(0,2]$

4. $f(x) = \sin \pi x$

$f(x) = \sin(\pi x) \qquad f(0) = \sin 0 = 0$

$f'(x) = \cos(\pi x) \cdot \pi \qquad f'(0) = \pi \cos(0) = \pi$

$= \pi \cos(\pi x)$

$f''(x) = -\pi \sin(\pi x) \cdot \pi \qquad f''(0) = -\pi^2 \sin(0) = 0$

$= -\pi^2 \sin(\pi x)$

$f'''(x) = -\pi^2 \cos(\pi x) \cdot \pi \qquad f'''(0) = -\pi^3 \cos 0 = -\pi^3$

$= -\pi^3 \cos(\pi x)$

$f^{(4)}(x) = +\pi^3 \sin(\pi x) \cdot \pi \qquad f^{(4)}(0) = \pi^4 \sin(0) = 0$

$= \pi^4 \sin(\pi x)$

$= 0 + \pi x + \frac{0x^2}{2!} - \frac{\pi^3 x^3}{3!} + \frac{0x^4}{4!} + \frac{\pi^5 x^5}{5!} + \dots$

$= \pi x - \frac{\pi^3 x^3}{3!} + \frac{\pi^5 x^5}{5!} - \frac{\pi^7 x^7}{7!} + \dots$

$\sum_{n=0}^{\infty} \frac{(-1)^n (\pi x)^{2n+1}}{(2n+1)!}$

$\sum_{n=0}^{\infty} \frac{(-1)^n (\pi x)^{2n+1}}{(2n+1)!}$

Interval of Convergence $(-\infty, \infty)$

5. $f(x) = e^{3x^2}$

$f(x) = e^{3x^2}$

$f'(x) = e^{3x^2} \cdot 6x$

$= 6x e^{3x^2}$

$f''(x) = 6x \cdot e^{3x^2} \cdot 6x + e^{3x^2} \cdot 6$

$= 36x^2 e^{3x^2} + 6e^{3x^2}$

$f'''(x) = 36x^2 \cdot e^{3x^2} \cdot 6x + e^{3x^2} \cdot 72x + 6e^{3x^2} \cdot 6x$

there has to be an easier way!

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$\therefore e^{3x^2} = 1 + 3x^2 + \frac{(3x^2)^2}{2!} + \frac{(3x^2)^3}{3!} + \frac{(3x^2)^4}{4!} + \dots$

$= 1 + 3x^2 + \frac{9x^4}{2} + \frac{27x^6}{3!} + \frac{81x^8}{4!} + \dots$

$\sum_{n=0}^{\infty} \frac{3^n x^{2n}}{n!}$

$(-\infty, \infty)$