AP Calculus BC
Maclaurin Series

Name Homework Guide
Power Series Day 3

MEMORIZE THESE MACLAURIN SERIES

f(x)	Maclaurin Series	General Term
e^x		
$\sin x$		
$\cos x$		
$\ln(1+x)$		
$\frac{1}{1-x}$		
$\frac{1}{1+x}$		
$\tan^{-1} x$		

1. Write out the first four terms of the Maclaurin of f(x) if
   f(0) = 2, f'(0) = 3, f''(0) = 4, f'''(0) = 12

$\frac{120}{4\cdot3\cdot2\cdot1} = \frac{10}{2}$

2-5: Find the Maclaurin series for f(x) using the definition of a Maclaurin series. [Assume that f has a power series expansion.] Also find the associated radius of convergence.

2. $f(x) = (1-x)^{-2}$

$f'(x) = -2(1-x)^{-3}(-1) = 2(1-x)^{-3}$

$f''(x) = -6(1-x)^{-4}(-1) = 6(1-x)^{-4}$

$f^3(x) = -24(1-x)^{-5}(-1) = 24(1-x)^{-5}$

$f^4(x) = -120(1-x)^{-6}(-1) = 120(1-x)^{-6}$

$f(0) = \frac{1}{(1-0)^2} = 1$

$f'(0) = \frac{2}{(1-0)^3} = 2$

$f^2(0) = \frac{6}{(1-0)^4} = 6$

$f^3(0) = 24/(1-0)^5 = 24$

$f^4(0) = 120/(1-0)^6 = 120$

$M(x) = \frac{1}{0!}(x-0)^0 + \frac{2}{1!}(x-0)^1 + \frac{6}{2!}(x-0)^2 + \frac{24}{3!}(x-0)^3 + \frac{120}{4!}(x-0)^4 + \ldots$

$M(x) = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + \ldots = \sum_{n=0}^{\infty} (n+1)x^n$

n=0 n=1 n=2 n=3 n=4

OR n=1 n=2 n=3 n=4 n=5 OR $\sum_{n=1}^{\infty} n \cdot x^{n-1}$

$R = \lim_{n\to\infty} \left| \frac{(n+2)x^{n+1}}{n+1\, x^n} \right| = \lim_{n\to\infty} \left| \frac{\cancel{x^n} x^1 (n+2)}{\cancel{x^n}(n+1)} \right|$

$R = |x| \lim_{n\to\infty} \left| \frac{n+2}{n+1} \right|$

$= |x|(1) < 1$

I.O.C = $-1, 1$

R.O.C = 1

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3. $f(x)=\ln(1+x)$

4. $f(x)=\sin \pi x$

$f'(x)=\pi\cos\pi x$

$f''(x)=-\pi^2\sin(\pi x)$

$f^3(x)=-\pi^3\cos(\pi x)$

$f^4(x)=\pi^4\sin(\pi x)$

$f^5(x)=\pi^5\cos(\pi x)$

$f(0)=\sin(0)=0$

$f'(0)=\pi\cos(0)=\pi$

$f^2(0)=-\pi^2\sin(0)=0$

$f^3(0)=-\pi^3\cos(0)=-\pi^3$

$f^4(0)=\pi^4\sin(0)=0$

$f^5(0)=\pi^5\cos(0)=\pi^5$

$$M(x)=\frac{0}{0!}(x-0)^0+\frac{\pi}{1!}(x-0)^1+\frac{0}{2!}(x-0)^2-\frac{\pi^3}{3!}(x-0)^3+\frac{0}{4!}(x-0)^4+\frac{\pi^5}{5!}(x-0)^5$$

$$M(x)=\underset{n=0}{\frac{\pi}{1!}(x)^1}-\underset{n=1}{\frac{\pi^3}{3!}x^3}+\underset{n=2}{\frac{\pi^5}{5!}x^5}=\sum_{n=0}^{\infty}\frac{(-1)^n\pi^{2n+1}x^{2n+1}}{(2n+1)!}$$

$$R=\lim_{n\to\infty}\left|\frac{\pi^{2n+3}x^{2n+3}}{(2n+3)!}\cdot\frac{(2n+1)!}{x^{2n+1}\cdot\pi^{2n+1}}\right|$$

$$R=\lim_{n\to\infty}\left|\frac{\cancel{x^{2n}}x^3\cancel{\pi^{2n}}\pi^3\cancel{(2n+1)!}}{\cancel{x^{2n}}x^1\cancel{\pi^{2n}}\pi^1(2n+3)(2n+2)\cancel{(2n+1)!}}\right|$$

$$R=|x^2|\lim_{n\to\infty}\left|\frac{\pi^2}{(2n+3)(2n+2)}\right|$$

$$R=|x^2|(0)<1$$

I.O.C $(-\infty,\infty)$

R.O.C $=\infty$

5. $f(x)=e^{3x^2}$

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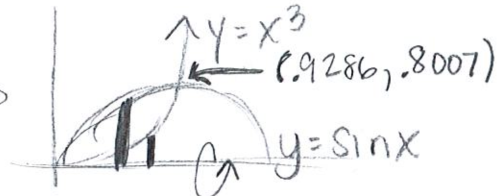
Maclaurin Series — Power Series Day 3

Review:

R1. **Calculator** What is the approximate volume of the solid obtained by revolving the region in the first quadrant enclosed by the curves $y = x^3$ and $y = \sin x$ about the x-axis?

a. 0.061π
b. 0.139π
c. 0.215π
d. 0.225π
e. 0.278π



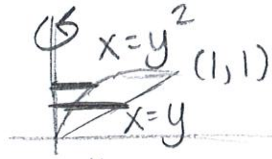
$\pi \int_{x_1}^{x_2} R^2 - r^2\,dx$

$\pi \int_0^{.9286} (\sin x - 0)^2 - (x^3 - 0)^2\,dx$

$\pi[.139427]$

R2. **Calculator** The volume of the solid generated by revolving the region bounded by the graphs of $y = \sqrt{x}$ and $y = x$ about the y-axis is

a. $\frac{2\pi}{15}$
b. $\frac{\pi}{6}$
c. $\frac{2\pi}{3}$
d. $\frac{16\pi}{15}$
e. $\frac{56\pi}{15}$



$\pi \int_{y_1}^{y_2} R^2 - r^2\,dy$

$\pi \int_0^1 (y-0)^2 - (y^2-0)^2\,dy$

$\pi(.1333)$

$\pi\left(\frac{2}{15}\right)$

R3. **Calculator** What is the approximate slope of the tangent to the curve $x^3 + y^3 = xy$ at x=1?

a. −2.420
b. −1.325
c. −1.014
d. −0.698
e. 0.267

Tangent line

$\boxed{1}$ Point $(1, -1.3247)$

$1 + y^3 = 1(y)$

$y^3 - y + 1 = 0$

Find zero

$Y_1 = x^3 - x + 1$

$x = -1.3247$

$y =$

R4. **Calculator** Given $f(x) = x^2e^x$, what is the approximate value of $f(1.1)$, if you use the tangent line to the graph of f at x=1?

a. 3.534
b. 3.635
c. 7.055
d. 8.155
e. 10.244

$\boxed{1}$ Point $f(1) = 1e^1$

$(1, e)$

$\boxed{2}$ Slope $f'(x) = x^2e^x + e^x(2x)$

$f'(1) = e^1 + 2e^1$

$f'(1) = 3e^1$

Answers:

1. $f(x) = 2 + 3x + 2x^2 + 2x^3$
2. $M(x) = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + ... = \sum_{n=0}^{\infty}(n+1)x^n$ or $\sum_{n=1}^{\infty} nx^{n-1}$ R.O.C =`1
3. $M(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} .... = \sum_{n=0}^{\infty}\frac{(-1)^n x^{n+1}}{n+1}$ R.O.C =`1
4. $M(x) = \pi x - \frac{\pi^3x^3}{3!} + \frac{\pi^5x^5}{5!} - \frac{\pi^7x^7}{7!} .... = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)!}$ R.O.C= ∞
5. $M(x) = 1 + 3x^2 + \frac{9x^4}{2} + \frac{27x^6}{3!} + \frac{81x^8}{4!} .... = \sum_{n=0}^{\infty}\frac{3^n x^{2n}}{n!}$ R.O.C= ∞

R1. B R2. A R3. C R4. A

$y - e = 3e(x-1)$

$y = 3e(x-1) + e$

$y(1.1) = 3e(.1) + e$

$y(1.1) = 3.5337$

$\boxed{2}$ Slope

$\frac{d}{dx}[x^3] + \frac{d}{dx}[y^3] = \frac{d}{dx}[xy]$

$3x^2 + 3y^2\frac{dy}{dx} = x\frac{dy}{dx} + y(1)$

$3y^2\frac{dy}{dx} - x\frac{dy}{dx} = y - 3x^2$

$\frac{dy}{dx} = \frac{y - 3x^2}{3y^2 - x}$

$\frac{dy}{dx} = \frac{(-1.3247) - 3(1)^2}{3(-1.3247)^2 - 1}$

$\frac{dy}{dx} = -1.014118$