AP Calculus BC

Radius and Interval of Convergence

Name Homework Guide

Day 2 Power Series

Find the radius of convergence and the interval of convergence of the series.

1. $\sum_{n=1}^{\infty}(-1)^n n x^n$ $\quad R=1 \quad I=(-1,1)$

$\lim_{n\to\infty}\frac{(n+1)x^{n+1}}{nx^n} = \lim_{n\to\infty}\frac{\cancel{x^n}x^1(n+1)}{\cancel{x^n}n} = |x|\lim_{n\to\infty}\frac{n+1}{n} = |x|(1)$

$|x|(1)<1$

$x<1$ & $x>-1$

$\boxed{\text{Check} -1}$ $\sum_{n=1}^{\infty}(-1)^n n(-1)^n = \sum_{n=1}^{\infty} n$ ∴ diverges by divergence $\lim_{n\to\infty} n = \infty \neq 0$.

$\boxed{\text{Check } 1}$ $\sum_{n=1}^{\infty}(-1)^n n(1)^n = \sum_{n=1}^{\infty}(-1)^n n$ ∴ diverges by divergence

let $n=$ even $\lim_{n\to\infty} n=\infty$ let $n=$ odd $\lim_{n\to\infty} -n=-\infty$ $\lim_{n\to\infty} =$ dne $\neq 0$

2. $\sum_{n=1}^{\infty}\frac{(-1)^n x^n}{\sqrt[3]{n}}$

3. $\sum_{n=1}^{\infty}\frac{x^n}{2n-1}$ $\quad R=1 \quad I=[-1,1)$

$\lim_{n\to\infty}\frac{x^{n+1}}{2(n+1)-1}\cdot\frac{(2n-1)}{x^n} = \lim_{n\to\infty}\frac{\cancel{x^n}x^1(2n-1)}{\cancel{x^n}(2n+1)} = |x|\lim_{n\to\infty}\frac{2n-1}{2n+1}$

$|x|(1)<1$

$x<1$ & $x>-1$ $\boxed{\text{Check}(-1)}$ $\sum_{n=1}^{\infty}\frac{(-1)^n}{2n-1}$ converges by alt. b.c. alternating & $\lim_{n\to\infty}\frac{1}{2n-1}=0$

$\boxed{\text{check } 1}$ $\sum_{n=1}^{\infty}\frac{(1)^n}{2n-1} = \sum_{n=1}^{\infty}\frac{1}{2n-1}$ $\quad b_n=\frac{1}{n}$ $\quad$ diverges by limit comparison

$\lim_{n\to\infty}\frac{1}{2n-1}\cdot\frac{n}{1}=\frac{1}{2}>0$ & $\sum_{n=1}^{\infty}\frac{1}{n}$ diverge by p-series $p=1\leq 1$

4. $\sum_{n=1}^{\infty}\frac{x^n}{n!}$

5. $\sum_{n=1}^{\infty}n^n x^n$ $\quad R=0 \quad I=\{0\}$

$\lim_{n\to\infty}\frac{(n+1)^{n+1}x^{n+1}}{n^n x^n} = \lim_{n\to\infty}\frac{(n+1)^n(n+1)^1\cancel{x^n}x^1}{n^n\cancel{x^n}}$

$|x|\lim_{n\to\infty}\left(\frac{n+1}{n}\right)^n(n+1)$

$|x|(1)^n(\infty)$

$\underbrace{|x|\infty<1}_{\text{only true when } x=0}$

6. $\sum_{n=1}^{\infty}(-1)^n\frac{n^2x^n}{2^n}$

7. $\sum_{n=1}^{\infty}\frac{(-3)^n}{n\sqrt{n}}x^n$ $\quad R=\frac{1}{3} \quad I=[-\frac{1}{3},\frac{1}{3}]$

$\lim_{n\to\infty}\frac{3^{n+1}x^{n+1}}{(n+1)\sqrt{n+1}}\cdot\frac{n\sqrt{n}}{x^n\cdot 3^n} = \lim_{n\to\infty}\frac{\cancel{3^n}3^1\cancel{x^n}x^1 n\sqrt{n}}{\cancel{3^n}\cancel{x^n}(n+1)\sqrt{n+1}}$

$|x|\lim_{n\to\infty}\frac{3n\sqrt{n}}{(n+1)\sqrt{n+1}}$ $\quad |x|(3)<1$ $\quad |x|<\frac{1}{3}$ $\quad x<\frac{1}{3}$ $\quad x>-\frac{1}{3}$

$\boxed{\text{Check}(-\frac{1}{3})}$ $\sum_{n=1}^{\infty}\frac{(-3)^n(-\frac{1}{3})^n}{n\sqrt{n}} = \sum_{n=1}^{\infty}\frac{1}{n\sqrt{n}}$ converges by p-series $p=\frac{3}{2}>1$

$\boxed{\text{Check}(\frac{1}{3})}$ $\sum_{n=1}^{\infty}\frac{(-3)^n(\frac{1}{3})^n}{n\sqrt{n}} = \sum_{n=1}^{\infty}\frac{(-1)^n}{n\sqrt{n}}$ alternating & $\lim_{n\to\infty}\frac{1}{n\sqrt{n}}=0$

Converges by alternating

8. $\sum_{n=1}^{\infty}(-1)^n\frac{x^n}{4^n\ln n}$

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9. $\sum_{n=1}^{\infty} \frac{3^n (x+4)^n}{\sqrt{n}}$ $R = \frac{1}{3}$ $I = \left[\frac{-13}{3}, \frac{-11}{3}\right)$

$\lim_{n\to\infty} \frac{3^{n+1}(x+4)^{n+1}}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{3^n (x+4)^n} = \lim_{n\to\infty} \frac{\cancel{3^n}\, 3^1 \cancel{(x+4)^n} (x+4)^1 \sqrt{n}}{\cancel{3^n} \cancel{(x+4)^n} \sqrt{n+1}}$

$|x+4| \lim_{n\to\infty} \frac{3\sqrt{n}}{\sqrt{n+1}}$

$|x+4|(3) < 1$

$|x+4| < 1/3$

$x+4 < \frac{1}{3}$ $x+4 > -\frac{1}{3}$

$x + \frac{12}{3} < \frac{1}{3}$ $x + \frac{12}{3} > -\frac{1}{3}$

$x < -11/3$ $x > -13/3$

$\boxed{\text{Check}\left(-\frac{13}{3}\right)}$ $\sum_{n=1}^{\infty} \frac{3^n\left(-\frac{13}{3}+4\right)^n}{\sqrt{n}}$

$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ is alternating & $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$ $\therefore$ converges by Alternating

$\boxed{\text{Check}\left(-\frac{11}{3}\right)}$ $\sum_{n=1}^{\infty} \frac{3^n\left(-\frac{11}{3}+4\right)^n}{\sqrt{n}}$

$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges by p-series $p = \frac{1}{2} \le 1$.

10. $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^2+1}$

11. $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^n}$ $R = \infty$ $I = (-\infty, \infty)$

$\lim_{n\to\infty} \frac{(x-2)^{n+1}}{(n+1)^{n+1}} \cdot \frac{n^n}{(x-2)^n} = \lim_{n\to\infty} \frac{\cancel{(x-2)^n}(x-2)^1 n^n}{(n+1)^n (n+1)^1 \cancel{(x-2)^n}}$

$|x-2| \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n \cdot \frac{1}{n+1}$

$|x-2|(1) \cdot \frac{1}{\infty}$

$\underbrace{|x-2|(0) < 1}_{\text{Always true}}$

12. $\sum_{n=1}^{\infty} n!(2x-1)^n$

Review:

R1. Which of the following series are convergent?

C I. $12 - 8 + \frac{16}{3} - \frac{32}{9} + \ldots$ $R = -2/3$ Converges by geometric

d II. $5 + \frac{5\sqrt{2}}{2} + \frac{5\sqrt{3}}{3} + \frac{5}{2} + \sqrt{5} + \frac{5\sqrt{6}}{6} + \ldots$ $\sum_{n=1}^{\infty} \frac{5\sqrt{n}}{n}$ diverges p-series

d III. $8 + 20 + 50 + 125 + \ldots$ way to big

a. I only b. II only c. III only
d. I & II e. II & III

R2. The $\lim_{h\to 0} \frac{\ln(x-3+h) - \ln(x-3)}{h}$ .Formal def der. Asking for $f'(x) =$

a. $\ln(x+3)$ b. $\ln(x-3)$ c. $\frac{1}{\ln(x-3)}$

d. $\frac{1}{x+3}$ e. $\frac{1}{x-3}$

Given $f(x) = \ln(x-3)$

$f'(x) = \frac{1}{x-3} \cdot (1) = \frac{1}{x-3}$

Answers:

1. $R=1$ $I.O.C = (-1,1)$
2. $R=1$ $I.O.C = (-1,1]$
3. $R=1$ $I.O.C = [-1,1)$
4. $R=\infty$ $I.O.C = (-\infty,\infty)$
5. $R=0$ $I.O.C = \{0\}$
6. $R=2$ $I.O.C = (-2,2)$
7. $R=\frac{1}{3}$ $I.O.C = \left[-\frac{1}{3}, \frac{1}{3}\right]$
8. $R=4$ $I.O.C = (-4,4]$
9. $R=\frac{1}{3}$ $I.O.C = \left[-\frac{13}{3}, -\frac{11}{3}\right)$
10. $R=1$ $I.O.C = [1,3]$
11. $R=\infty$ $I.O.C = (-\infty,\infty)$
12. $R=0$ $I.O.C = \left\{\frac{1}{2}\right\}$

R1. A

R2. E