Know: $\frac{1}{1+x} = 1 - x + x^2 - x^3 + \ldots \sum_{n=0}^{\infty} (-1)^n x^n$

AP Calculus BC

Power Series Know: $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots \sum_{n=0}^{\infty} x^n$

Name Homework Guide

Day 1 Power Series

Expand the function in a power series with center c = 0 and determine the set of x for which the expansion is valid.

K: $\frac{1}{1-X} = 1 + X + X^2 + \ldots + = \sum_{n=0}^{\infty} X^n$

1. $f(x) = \frac{1}{1-(3x)}$

$\frac{1}{1-(3x)} = 1 + (3x) + (3x)^2 + (3x)^3 + \ldots = \sum_{n=0}^{\infty} (3x)^n$

$\underline{OR}$ $\sum_{n=0}^{\infty} 3^n x^n$

$|3x| < 1$

$3x < 1$ and $3x > -1$

$x < 1/3$ and $x > -\frac{1}{3}$

$\left(-\frac{1}{3}, \frac{1}{3}\right)$

2. $f(x) = \frac{1}{1+3x}$

3. $f(x) = \frac{1}{3-x}$

4. $f(x) = \frac{1}{4+3x} = \frac{1}{4\left[1+\frac{3}{4}x\right]}$

$\frac{1}{4}\left[\frac{1}{1+\left(\frac{3}{4}x\right)}\right] = \frac{1}{4}\cdot 1 - \frac{1}{4}\left(\frac{3}{4}x\right) + \frac{1}{4}\left(\frac{3}{4}x\right)^2 - \frac{1}{4}\left(\frac{3}{4}x\right)^3$

$= \sum_{n=0}^{\infty} (-1)^n \cdot \frac{1}{4}\left(\frac{3}{4}x\right)^n = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot (3x)^n}{4^1 \cdot 4^n}$

$= \sum_{n=0}^{\infty} \frac{(-1)^n (3x)^n}{4^{n+1}}$

$\left|\frac{3}{4}x\right| < 1$

$\frac{3}{4}x < 1$ & $\frac{3}{4}x > -1$

$x < \frac{4}{3}$ $x > -\frac{4}{3}$ $\left(-\frac{4}{3}, \frac{4}{3}\right)$

5. $f(x) = \frac{1}{1+x^9}$

6. $f(x) = \frac{1}{5-x^2} = \frac{1}{5\left[1-\frac{x^2}{5}\right]}$

$\frac{1}{5}\cdot 1 + \frac{1}{5}\left(\frac{x^2}{5}\right) + \frac{1}{5}\left(\frac{x^2}{5}\right)^2 + \frac{1}{5}\left(\frac{x^2}{5}\right)^3 + \ldots$

$= \sum_{n=0}^{\infty} \frac{1}{5}\left(\frac{x^2}{5}\right)^n = \sum_{n=0}^{\infty} \frac{x^{2n}}{5^1 \cdot 5^n} = \sum_{n=0}^{\infty} \frac{x^{2n}}{5^{n+1}}$

$\left|\frac{x^2}{5}\right| < 1$

$\frac{x^2}{5} < 1$ $\frac{x^2}{5} > -1$

$\sqrt{x^2} < \sqrt{5}$ $\cancel{\sqrt{x^2} > \sqrt{5}}$ garbage

$x < \sqrt{5}$ &

$x > -\sqrt{5}$

$(-\sqrt{5}, \sqrt{5})$

AP Calculus BC
Power Series

Name______________________________
Day 1 Power Series

7. $f(x) = \dfrac{1}{1+3x^7}$

Concept Review: Use the ratio test to prove absolute convergence or divergence for each series.

8. $\sum_{n=1}^{\infty} \frac{2^n}{n}$

$R = \lim_{n\to\infty} \frac{2^{n+1}}{n+1} \cdot \frac{n}{2^n} = \lim_{n\to\infty} \frac{\cancel{2^n} \cdot 2^1 \cdot n}{\cancel{2^n}(n+1)} = 2$

diverges by Ratio test
$|R| = 2 > 1$

9. $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ $b_n = \frac{n}{n^2} = \frac{1}{n}$

$\lim_{n\to\infty} \frac{n}{n^2+1} \cdot \frac{n}{1} = 1 > 0$

& $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges by p-series $p = 1 \leq 1$.

div

10. $\sum_{n=1}^{\infty} \frac{e^n}{n!}$

11. $\sum_{n=1}^{\infty} \frac{n!}{n^4}$

AP Calculus BC
Power Series
Review:

R1. $\int_{-2}^{2} 3e^{-x}dx$

$\left.\frac{3e^{-x}}{-1}\right]_{-2}^{2}$

$\left.-3e^{-x}\right]_{-2}^{2}$

$-3e^{-2}+3e^{2}$

$3(e^{2}-e^{-2})$

a. $-3e^{-2}$
b. $-3e^{2}$
c. $6(1-e^{-2})$
d. $3(e^{2}-e^{-2})$
e. $3(e^{-2}-e^{2})$

Name______________________________
Day 1 Power Series

R2. If $f(x)=x^3+3x^2+cx+4$ has a horizontal tangent and a point of inflection at the same value of x, what is the value of c?
a. 0
b. 1
c. −1
d. −3
e. 3

$f'(x)=3x^2+6x+C$
$0=3(-1)^2+6(-1)+C$
$0=3-6+C$
$0=-3+C$
$C=3$

$f''(x)=6x+6$
$0=6x+6$
$x=-1$

R3. Find $\frac{dy}{dx}$ if $\tan y=(x-y)^2$

a. $\frac{dy}{dx}=\frac{2(x-y)}{\sec^2 y+2(x-y)}$
b. $\frac{dy}{dx}=\frac{2(x-y)}{\sec^2 y}$
c. $\frac{dy}{dx}=\frac{\sec^2 y-2(x-y)}{-2(x-y)}$
d. $\frac{dy}{dx}=\frac{1}{1+\sec^2 y}$
e. $\frac{dy}{dx}=1+\sec^2 y$

R4. Find $\frac{dy}{dx}$ if $3^{(4-x^2)}$

a. $\frac{dy}{dx}=(\ln 3)3^{(4-x^2)}$
b. $\frac{dy}{dx}=-2x(\ln 3)3^{(4-x^2)}$
c. $\frac{dy}{dx}=-2x(4-x^2)\ln(3)$
d. $\frac{dy}{dx}=-2x3^{(4-x^2)}$
e. $\frac{dy}{dx}=(4-x^2)3^{(4-x^2)}$

$\frac{d}{dx}[b^{AT}]$
$=b^{AT}\cdot \ln b\cdot \frac{d}{dx}[AT]$

$3^{4-x^2}\cdot \ln 3\cdot(-2x)$

$\frac{d}{dx}[\tan y]=\frac{d}{dx}[(x-y)^2]$

$\sec^2 y\cdot\frac{dy}{dx}=2(x-y)'\cdot\left[1-(1)\frac{dy}{dx}\right]$

$\sec^2 y\frac{dy}{dx}=(2x-2y)\left(1-\frac{dy}{dx}\right)$

$\sec^2 y\frac{dy}{dx}=2x-2x\frac{dy}{dx}-2y+2y\frac{dy}{dx}$

$\sec^2 y\frac{dy}{dx}+2x\frac{dy}{dx}-2y\frac{dy}{dx}=2x-2y$

$\frac{dy}{dx}\left[\sec^2 y+2x-2y\right]=2x-2y$

$\frac{dy}{dx}=\frac{2x-2y}{\sec^2 y+2x-2y}=\frac{2x-2y}{\sec^2 y+2(x-y)}$

Answers:

1. $\sum_{n=0}^{\infty}(3x)^n \quad \left(-\frac{1}{3},\frac{1}{3}\right)$
2. $\sum_{n=0}^{\infty}(-1)^n x^n \quad \left(-\frac{1}{3},\frac{1}{3}\right)$
3. $\sum_{n=0}^{\infty}\frac{x^n}{3^{n+1}} \quad (-3,3)$
4. $\sum_{n=0}^{\infty}\frac{(-1)^n(3x)^n}{4^{n+1}} \quad \left(-\frac{4}{3},\frac{4}{3}\right)$
5. $\sum_{n=0}^{\infty}(-1)^n x^{9n} \quad (-1,1)$
6. $\sum_{n=0}^{\infty}\frac{x^{2n}}{5^{n+1}} \quad (-\sqrt{5},\sqrt{5})$
7. $\sum_{n=0}^{\infty}(-1)^n 3^n x^{7n} \quad \left(\sqrt[7]{-\frac{1}{3}},\sqrt[7]{\frac{1}{3}}\right)$
8. Diverges Ratio
9. Diverges Limit Comparison
10. Converges Ratio
11. Diverges Ratio

R1. D
R2. E
R3. A
R4. B