~~Non~~-Calculator

Particle Motion #1

Name_____

Date_____Pd_____

For $0 \le t \le 6$, a particle is moving along the $x-axis$. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin\left(e^{\frac{t}{4}}\right)+1$. The acceleration of the particle is given by $a(t) = \frac{1}{2}\cos\left(e^{\frac{t}{4}}\right)$ and $x(0) = 2$. $a(t) = \frac{1}{2}e^{\frac{t}{4}} \cdot \cos(e^{t/4})$

a.) Is the speed of the particle increasing or decreasing at time $t = 5.5$? Give a reason for your answer.

$v(5.5) = 2\sin(e^{\frac{5.5}{4}})+1 = -.453$ $a(5.5) = \frac{1}{2}e^{\frac{5.5}{4}}\cos(e^{\frac{5.5}{4}}) = -1.359$

The speed of the particle is increasing because $v(5.5)$ & $a(5.5)$ have the same sign. +2: conclusion w/reason

b.) Find the average velocity of the particle for the time period $0 \le t \le 6$.

$\frac{1}{6-0}\int_0^6 v(t)dt = 1.949$

+1: Integral +1: answer

c.) Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.

$\int_0^6 |v(t)|dt = 12.573$

+1: Integral +1: answer

d.) For $0 \le t \le 6$, the particle changes direction exactly once. Find the position of the particle at the time.

+1: Considers $v(t)=0$

$v(t) = 0$?

$t = 5.195$ or $t = 5.196$

$t = 5.1955223$

Where you are = Where you start + how far you go

$x(5.195) = x(0) + \int_0^{5.195} v(t)dt$

$x(5.195) = 2 + 12.13477$ +1: Integral

$x(5.195) = 14.134$ or 14.135 } +1: answer

Non-Calculator
Particle Motion #4

Name _____
Date _____ pd ___

For $0 \le t \le 12$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = \cos\left(\frac{\pi}{6}t\right)$. The particle is at position $x = -2$ at time $t = 0$.

a.) For $0 \le t \le 12$, when is the particle moving to the left?

Moving to left when $v(t) < 0$.

+1: considers $v(t) = 0$

$v(t) = 0$

$\cos\left(\frac{\pi}{6}t\right) = 0$

$\cos t = 0 \quad t = \frac{\pi}{2}, \frac{3\pi}{2}$

$\frac{\pi}{6}t = \frac{\pi}{2} \cdot \frac{6}{\pi} \quad \frac{\pi}{6}t = \frac{3\pi}{2} \cdot \frac{6}{\pi}$

$t = 3 \quad t = 9$

+ 3 − 9 + $v(t)$

$v(1) = \cos\frac{\pi}{6} = +$

$v(4) = \cos\frac{2\pi}{3} = -$

$v(10) = \cos\left(\frac{5\pi}{3}\right) =$

+1: Interval

Moving left on $(3, 9)$

b.) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to $t = 6$.

total distance $= \int_0^6 |v(t)|\,dt = \int_0^3 v(t)\,dt - \int_3^6 v(t)\,dt$

+1: answer

c.) Find the acceleration of the particle at time t. Is the speed increasing, decreasing, or neither at time $t = 4$? Explain your reasoning.

$v(t) = \cos\left(\frac{\pi}{6}t\right)$

$v'(t) = a(t) = -\frac{\pi}{6}\sin\left(\frac{\pi}{6}t\right)$ +1: $a(t)$

$v(4) =$ negative

$a(4) = -\frac{\pi}{6}\sin\left(\frac{2\pi}{3}\right) =$ negative

The speed of the particle is increasing at $t = 4$ because $v(4)$ & $a(4)$ have the same sign. +2: conclusion with reason

d.) Find the position of the particle at time $t = 4$.

$p(0) = -2$

$p(4) = p(0) + \frac{6}{\pi}\int_0^4 \cos\left(\frac{\pi}{6}t\right)dt$

+1: uses initial condition

$u = \frac{\pi}{6}t \quad du = \frac{\pi}{6}dt \quad \frac{6}{\pi}du = dt$

$u(4) = \frac{4\pi}{6} = \frac{2\pi}{3} \quad u(0) = \frac{(0)\pi}{6} = 0$

$p(4) = -2 + \frac{6}{\pi}\sin u \Big|_0^{\frac{2\pi}{3}}$ +1: antiderivative

$p(4) = -2 + \frac{6}{\pi}\sin\frac{2\pi}{3} - \sin(0)$

$p(4) = -2 + \frac{6\sqrt{3}}{\pi 2} = -2 + \frac{3\sqrt{3}}{\pi}$

+1: answer