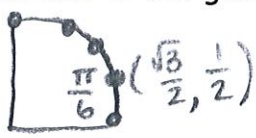
AP Calculus BC Name________________________

Derivatives of Polar Functions P, P, & V Day 9

$$\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{\frac{dr}{d\theta}\sin\theta+r\cos\theta}{\frac{dr}{d\theta}\cos\theta-r\sin\theta}=\frac{r'(\theta)\sin\theta+r\cos\theta}{r'(\theta)\cos\theta-r\sin\theta}$$

Find the slope of the tangent line to the given polar curve at the point specified by the value of θ.

$r' = 2\cos\theta$



1. $r = 2\sin\theta, \quad \theta = \frac{\pi}{6}$

$$\frac{dy}{dx} = \frac{(2\cos\theta)\sin\theta + (2\sin\theta)\cos\theta}{(2\cos\theta)\cos\theta - (2\sin\theta)\sin\theta}$$

$$\frac{dy}{dx} = \frac{\cancel{2}\left(\frac{\sqrt3}{\cancel{2}}\right)\left(\frac{1}{2}\right) + \cancel{2}\left(\frac{1}{\cancel{2}}\right)\left(\frac{\sqrt3}{2}\right)}{\cancel{2}\left(\frac{\sqrt3}{\cancel{2}}\right)\left(\frac{\sqrt3}{2}\right) - \cancel{2}\left(\frac{1}{\cancel{2}}\right)\left(\frac{1}{2}\right)}$$

$$\frac{dy}{dx} = \frac{\frac{\sqrt3}{2} + \frac{\sqrt3}{2}}{\frac{3}{2} - \frac{1}{2}} = \frac{\frac{\cancel{2}\sqrt3}{\cancel{2}}}{\frac{2}{2}} = \boxed{\sqrt3}$$

$r' = -\cos\theta$

2. $r = 2 - \sin\theta, \quad \theta = \frac{\pi}{3}$

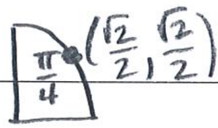
$$\frac{dy}{dx} = \frac{(-\cos\theta)\sin\theta + (2-\sin\theta)\cos\theta}{(-\cos\theta)\cos\theta - (2-\sin\theta)\sin\theta}$$

$$\frac{dy}{dx} = \frac{-\frac{1}{2}\left(\frac{\sqrt3}{2}\right) + \left(2 - \frac{\sqrt3}{2}\right)\left(\frac{1}{2}\right)}{-\frac{1}{2}\left(\frac{1}{2}\right) - \left(2 - \frac{\sqrt3}{2}\right)\left(\frac{\sqrt3}{2}\right)}$$

$$\frac{dy}{dx} = \frac{-\frac{\sqrt3}{4} + \frac{2}{2} - \frac{\sqrt3}{4}}{-\frac{1}{4} - \frac{2\sqrt3}{2} + \frac{3}{4}} = \frac{-\frac{2\sqrt3}{4} + \frac{2\cdot2}{2\cdot2}}{-\frac{2\sqrt3\cdot2}{2\cdot2} + \frac{2}{4}}$$

$$\frac{dy}{dx} = \frac{\frac{-2\sqrt3+4}{\cancel{4}} \cdot \frac{\cancel{4}}{-4\sqrt3+2}}{\frac{-4\sqrt3+2}{\cancel{4}} \cdot \frac{\cancel{4}}{-4\sqrt3+2}} = \frac{-2\sqrt3+4}{-4\sqrt3+2} = \boxed{\frac{2-\sqrt3}{1-2\sqrt3}}$$

$r' = -2\sin\theta$



3. $r = 2\cos\theta, \quad \theta = \frac{\pi}{4}$

$$\frac{dy}{dx} = \frac{(-2\sin\theta)\sin\theta + 2\cos\theta\cos\theta}{(-2\sin\theta)\cos\theta - 2\cos\theta\sin\theta}$$

$$\frac{dy}{dx} = \frac{-\cancel{2}\left(\frac{\sqrt2}{\cancel{2}}\right)\left(\frac{\sqrt2}{2}\right) + \cancel{2}\left(\frac{\sqrt2}{\cancel{2}}\right)\left(\frac{\sqrt2}{2}\right)}{-\cancel{2}\left(\frac{\sqrt2}{\cancel{2}}\right)\left(\frac{\sqrt2}{2}\right) - \cancel{2}\left(\frac{\sqrt2}{\cancel{2}}\right)\left(\frac{\sqrt2}{2}\right)}$$

$$\frac{dy}{dx} = \frac{\frac{2}{2} + \frac{2}{2}}{\frac{2}{2} - \frac{2}{2}} = \frac{2}{0} = \boxed{\text{undefined}}$$

4. $r = 1 + 2\cos\theta, \quad \theta = \frac{\pi}{3}$ $\left(\frac{1}{2}, \frac{\sqrt3}{2}\right)$

$r' = -2\sin\theta$

$$\frac{dy}{dx} = \frac{(-2\sin\theta)\sin\theta + (1+2\cos\theta)\cos\theta}{(-2\sin\theta)\cos\theta - (1+2\cos\theta)\sin\theta}$$

$$\frac{dy}{dx} = \frac{-\cancel{2}\left(\frac{\sqrt3}{\cancel{2}}\right)\left(\frac{\sqrt3}{2}\right) + \left(1+2\left(\frac{1}{2}\right)\right)\left(\frac{1}{2}\right)}{-\cancel{2}\left(\frac{\sqrt3}{\cancel{2}}\right)\left(\frac{1}{2}\right) - \left(1+2\left(\frac{1}{2}\right)\right)\left(\frac{\sqrt3}{2}\right)}$$

$$\frac{dy}{dx} = \frac{-\frac{3}{2} + (1+1)\left(\frac{1}{2}\right)}{-\frac{\sqrt3}{2} - (1+1)\left(\frac{\sqrt3}{2}\right)} \quad \frac{-\frac{3}{2} + \frac{2}{2}}{-\frac{\sqrt3}{2} - \frac{2\sqrt3}{2}}$$

$$\frac{dy}{dx} = \frac{-\frac{1}{\cancel{2}} \cdot \frac{-\cancel{2}}{3\sqrt3}}{-\frac{3\sqrt3}{\cancel{2}} \cdot \frac{-\cancel{2}}{3\sqrt3}} = \frac{1\cdot\sqrt3}{3\sqrt3\cdot\sqrt3} = \boxed{\frac{\sqrt3}{9}}$$

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$\frac{\pi}{6}$ $\left(\frac{\sqrt3}{2},\frac12\right)$

Find the points on the given curve where the tangent line is horizontal or vertical.

5. $r=3\cos\theta$ $\quad r'=-3\sin\theta$

$$\frac{dy}{dx}=\frac{(-3\sin\theta)\sin\theta+(3\cos\theta)\cos\theta}{(-3\sin\theta)\cos\theta-(3\cos\theta)\sin\theta}$$

Horizontal Tangents

$-3\sin^2\theta+3\cos^2\theta=0$

$-3(1-\cos^2\theta)+3\cos^2\theta=0$

$-3+3\cos^2\theta+3\cos^2\theta=0$

$6\cos^2\theta=3$

$\cos^2\theta=\frac12$

$\cos\theta=\pm\sqrt{\frac12}$

$\cos\theta=\frac{\pm1}{\sqrt2}$

$\cos\theta=\pm\frac{\sqrt2}{2}$

$\theta=\frac{\pi}{4}$ OR $\frac{3\pi}{4}$

$R=3\cos\theta$

$R=3\cos\frac{\pi}{4}=\frac{3\sqrt2}{2}$

$R=3\cos\frac{3\pi}{4}=-\frac{3\sqrt2}{2}$

$\boxed{\left(\frac{3\sqrt2}{2},\frac{\pi}{4}\right)\quad\left(-\frac{3\sqrt2}{2},\frac{3\pi}{4}\right)}$

Vertical Tangents

$-3\sin\theta\cos\theta-3\sin\theta\cos\theta=0$

$-6\sin\theta\cos\theta=0$

$\sin\theta\cos\theta=0$

$\theta=0 \qquad \theta=\frac{\pi}{2}$

$r=3\cos(0) \qquad r=3\cos\frac{\pi}{2}$

$r=3 \qquad r=0$

$\boxed{(3,0)\qquad\left(0,\frac{\pi}{2}\right)}$

6. $r=1-\sin\theta$ $\quad r'=-\cos\theta$

$$\frac{dy}{dx}=\frac{(-\cos\theta)\sin\theta+(1-\sin\theta)\cos\theta}{(-\cos\theta)\cos\theta-(1-\sin\theta)\sin\theta}$$

Horizontal Tangents

$-\cos\theta\sin\theta+\cos\theta-\cos\theta\sin\theta=0$

$-2\cos\theta\sin\theta+\cos\theta=0$

$\cos\theta[-2\sin\theta+1]=0$

Graph

$\cos\theta=0 \qquad \sin\theta=\frac12$

$\theta=\frac{\pi}{2} \qquad \theta=\frac{\pi}{6}$

$r=1-\sin\frac{\pi}{2} \qquad r=1-\sin\frac{\pi}{6}$

$r=1-1=0 \qquad r=1-\frac12=\frac12$

$\boxed{\cancel{\left(0,\frac{\pi}{2}\right)}\qquad\left(\frac12,\frac{\pi}{6}\right)}$

$0,\frac{\pi}{2}$ twice not HT

$\left(\frac{\sqrt3}{2},-\frac12\right)$

vertical tangents

$-\cos^2\theta-\sin\theta+\sin^2\theta=0$

$-(1-\sin^2\theta)-\sin\theta+\sin^2\theta=0$

$-1+\sin^2\theta-\sin\theta+\sin^2\theta=0$

$2\sin^2\theta-\sin\theta-1=0$

$(2\sin\theta+1)(\sin\theta-1)$

$\sin\theta=-\frac12 \qquad \sin\theta=1$

$\theta=-\frac{\pi}{6} \qquad \theta=\frac{\pi}{2}$

$r=1-\sin\left(-\frac{\pi}{6}\right)$

$1--\frac12=\frac32$

$\boxed{\left(0,\frac{\pi}{2}\right)\quad\left(\frac32,-\frac{\pi}{6}\right)}$