AP Calculus BC

Name_____

Polar Coordinates

P, P, & V Day 8

1-2: Plot the point whose polar Coordinates are given. Then find two other pairs of polar coordinates of this point.

1. A.) $\left(2,\frac{\pi}{3}\right)$ B.) $\left(1,\frac{-3\pi}{4}\right)$ C.) $\left(-1,\frac{\pi}{2}\right)$

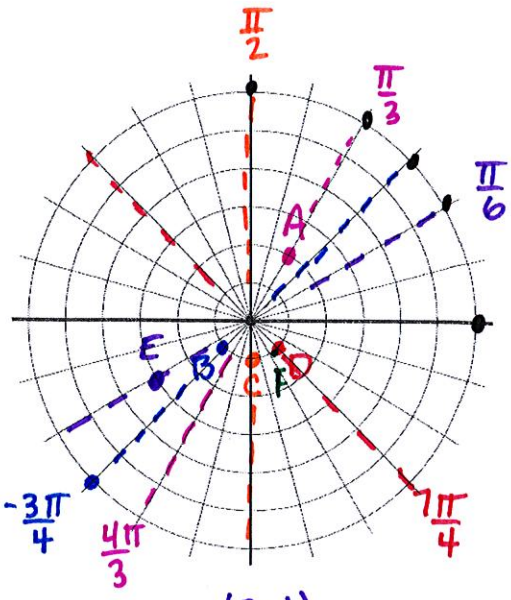
$\left(-2,\frac{4\pi}{3}\right)$ $\left(-1,\frac{\pi}{4}\right)$ $\left(1,\frac{3\pi}{2}\right)$

$\left(2,\frac{7\pi}{3}\right)$ $\left(1,\frac{5\pi}{4}\right)$ $\left(-1,\frac{-\pi}{2}\right)$

2. D.) $\left(1,\frac{7\pi}{4}\right)$ E.) $\left(-3,\frac{\pi}{6}\right)$ F.) $(-1,-1)$

$\left(-1,\frac{3\pi}{4}\right)$ $\left(3,\frac{7\pi}{6}\right)$ $(-1,-1+2\pi)$

$\left(1,\frac{15\pi}{4}\right)$ $\left(-3,\frac{-11\pi}{6}\right)$ $(1,-1+\pi)$



$\frac{-\pi}{4} \approx -.78$

$\frac{-\pi}{3} \approx -1.04$

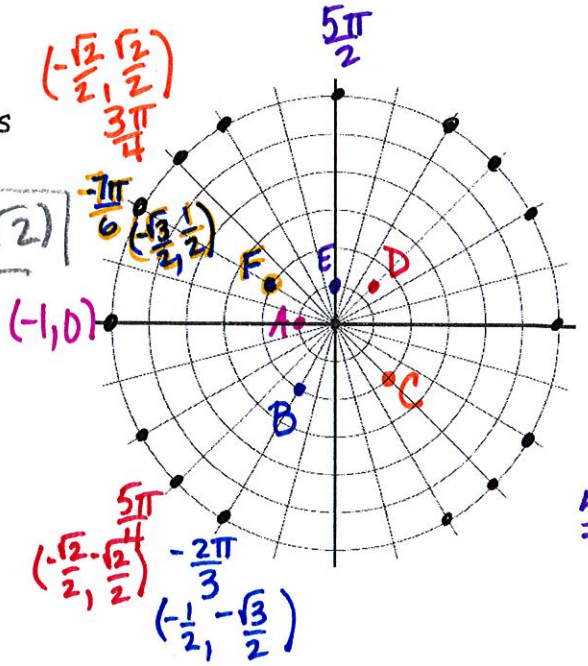
3-4: Plot the point whose polar coordinates are given. Then find the Cartesian coordinates of the point.

3. A.) $(1,\pi)$ B.) $\left(2,\frac{-2\pi}{3}\right)$ C.) $\left(-2,\frac{3\pi}{4}\right)$ $=(\sqrt{2},-\sqrt{2})$

$x=1\cos\pi$ $x=2\cos\frac{-2\pi}{3}$ $x=-2\left(\frac{-\sqrt{2}}{2}\right)$

$y=1\sin\pi$ $y=2\sin\frac{-2\pi}{3}$ $y=-2\left(\frac{\sqrt{2}}{2}\right)$

$(-1,0)$ $(-1,-\sqrt{3})$



$\sqrt{2} \approx 1.4$

$\frac{5\pi}{2} - 2\pi = \frac{\pi}{2}$

4. D.) $\left(-\sqrt{2},\frac{5\pi}{4}\right)$ E.) $\left(1,\frac{5\pi}{2}\right)$ F.) $\left(2,\frac{-7\pi}{6}\right)$

$x=-\sqrt{2}\left(-\frac{\sqrt{2}}{2}\right)$ $x=1(0)$ $x=2\left(-\frac{\sqrt{3}}{2}\right)$

$y=-\sqrt{2}\left(-\frac{\sqrt{2}}{2}\right)$ $y=1(1)$ $y=2\left(\frac{1}{2}\right)$

$(1,1)$ $(0,1)$ $(-\sqrt{3},1)$

5-6: The Cartesian coordinates of a point are given.

i.) Find the polar coordinates (r,θ) of the point, where $r>0$ and $0\le\theta\le 2\pi$.

ii.) Find the polar coordinates (r,θ) of the point, where $r<0$ and $0\le\theta\le 2\pi$.

5. A.) $(2,-2)$ Q4 B.) $(-1,\sqrt{3})$ Q3

$r=\sqrt{(2)^2+(-2)^2}$ $\left(2\sqrt{2},\frac{7\pi}{4}\right)$

$r=\sqrt{4+4}=\sqrt{8}$ $\left(-2\sqrt{2},\frac{3\pi}{4}\right)$

$r=2\sqrt{2}$

$\tan^{-1}\left(\frac{-2}{2}\right)=\tan^{-1}(-1)$

$\theta=-\frac{\pi}{4}$ (Q4)

$r=\sqrt{1+3}=2$

$\theta=\tan^{-1}(-\sqrt{3})$

$\theta=-\frac{\pi}{3}+\frac{3\pi}{3}$

$\left(2,\frac{2\pi}{3}\right)$ $\left(-2,\frac{5\pi}{3}\right)$

6. A.) $(3\sqrt{3},3)$ Q1

$r=\sqrt{27+9}=6$

$\theta=\tan^{-1}\left(\frac{3}{3\sqrt{3}}\right)$

$\theta=\tan^{-1}\left(\sqrt{3}/3\right)$

$\theta=\frac{\pi}{6}$

$\left(6,\frac{\pi}{6}\right)$ $\left(-6,\frac{7\pi}{6}\right)$

B.) $(1,-2)$ Q4

$r=\sqrt{1+4}=\sqrt{5}$

$\theta=\tan^{-1}\left(\frac{-2}{1}\right)\approx -63.4$

$(\sqrt{5},296.6^\circ)$

$(-\sqrt{5},116.6^\circ)$

$r^2 = x^2 + y^2 \quad \theta = \tan^{-1}\left(\frac{y}{x}\right)$

$x = r\cos\theta \quad y = r\sin\theta$

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7-12: Identify the curve by finding a Cartesian equation for the curve.

7. $r^2 = 5$

$\boxed{x^2 + y^2 = 5}$

8. $r = 4\sec\theta$

$r = \frac{4}{\cos\theta}$

$r\cos\theta = 4$

$\boxed{x = 4}$

9. $r = 2\cos\theta$

$r^2 = 2r\cos\theta$

$x^2 + y^2 = 2x$

$x^2 + 2x + 1 + y^2 = 0 + 1$

$\boxed{(x+1)^2 + y^2 = 1}$

10. $\theta = \frac{\pi}{3}$

$\tan^{-1}\left(\frac{y}{x}\right) = \frac{\pi}{3}$ (tan)

$\frac{y}{x} = \tan\frac{\pi}{3}$

$y = x\tan\frac{\pi}{3}$

$\boxed{y = \sqrt{3}x}$

11. $r^2\cos 2\theta = 1$

$R^2(\cos^2\theta - \sin^2\theta) = 1$

$R^2\cos^2\theta - R^2\sin^2\theta = 1$

$(R\cos\theta)^2 - (R\sin\theta)^2 = 1$

$\boxed{x^2 - y^2 = 1}$

12. $r = \tan\theta\sec\theta$

$r = \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta}$

$r = \frac{\sin\theta}{\cos^2\theta}$

$r \cdot r\cos^2\theta = r\sin\theta$

$(r\cos\theta)^2 = r\sin\theta$

$\boxed{x^2 = y}$

13-16: Find a polar equation for the curve represented by the given Cartesian equation.

13. $y = 2$

$r\sin\theta = 2$

$r = \frac{2}{\sin\theta}$

$\boxed{r = 2\csc\theta}$

14. $y = x$

$\frac{y}{x} = \frac{x}{x}$

$\frac{y}{x} = 1$

$\tan^{-1}\frac{y}{x} = \tan^{-1}(1)$

$\boxed{\theta = \frac{\pi}{4}}$

15. $y = 1 + 3x$

$r\sin\theta = 1 + 3r\cos\theta$

$r\sin\theta - 3r\cos\theta = 1$

$r(\sin\theta - 3\cos\theta) = 1$

$\boxed{r = \frac{1}{\sin\theta - 3\cos\theta}}$

16. $xy = 4$

$r\cos\theta\, r\sin\theta = 4$

$r^2\sin\theta\cos\theta = 4$

$r^2 = \frac{4}{\sin\theta\cos\theta}$

$r = \frac{\pm 2}{\sqrt{\sin\theta\cos\theta}}$