AP Calculus Name________________________

Parametric and Vectors P, P, & V Day 7

1. Find the equation of the tangent line to $x = 4\sin t,\ \ y = 2\cos t:\ \ at\ t = \frac{\pi}{2}$

1. Point: $x = 4\sin\frac{\pi}{2}$ $\quad 4(1) \quad 4$ $\qquad y = 2\cos\frac{\pi}{2}$ $\quad 2(0) \quad 0$

$(4,0)$

$\boxed{2.}$ Slope $= \dfrac{dy/dt}{dx/dt} = \dfrac{-2\sin\frac{\pi}{2}}{4\cos\frac{\pi}{2}} = \dfrac{-2}{0} =$ undefined

Vertical lines have undefined slopes $\boxed{x=4}$

2. Find all POINTS at which the tangent line to the curve is A.) Horizontal and B.) Vertical given $x = 2 - t,\ \ y = t^3 - 4t$.

A) Horizontal

$\frac{dy}{dt} = 0$

$3t^2 - 4 = 0$

$\sqrt{t^2} = \sqrt{\frac{4}{3}}$

$t = \pm\, 2/\sqrt{3} = \pm\frac{2\sqrt{3}}{3}$

$\left(2 - \frac{2\sqrt{3}}{3},\ \frac{8\cdot\cancel{3}\sqrt{3}}{\cancel{27}\,9} - \frac{4\cdot 2\sqrt{3}\cdot 3}{3\cdot 3}\right)$

$\left(2 + \frac{2\sqrt{3}}{3},\ -\frac{8\cdot\cancel{3}\sqrt{3}}{\cancel{27}\,9} + \frac{4\cdot 2\sqrt{3}\cdot 3}{\cancel{3}\,3}\right)$

$\boxed{\left(2 - \frac{2\sqrt{3}}{3},\ -\frac{16\sqrt{3}}{9}\right)\ \ \left(2 + \frac{2\sqrt{3}}{3},\ \frac{16\sqrt{3}}{9}\right)}$

B) Vertical

$\frac{dx}{dt} = 0$

$\cancel{-1 = 0}$

$\boxed{\text{none}}$

3. Determine find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$, and determine intervals of concavity for $x = t^2 + t,\ \ y = t^2 - t$.

$x'(t) = 2t + 1 \qquad x''(t) = 2$

$y'(t) = 2t - 1 \qquad y''(t) = 2$

$\boxed{\frac{dy}{dx} = \frac{2t+1}{2t-1}}$

$\frac{d^2y}{dx^2} = \frac{(2t+1)(2) - (2t-1)(2)}{(2t+1)^3} = \frac{\cancel{4t} + 2 - \cancel{4t} + 2}{(2t+1)^3}$

$\frac{d^2y}{dx^2} = \frac{4}{(2t+1)^3}$

Positive

Neg. $-\frac{1}{2}$ Pos

$2t + 1 = 0$

$-1/2$

$\boxed{\text{Concave up } (-\tfrac{1}{2}, \infty)}$

$\boxed{\text{Concave down } (-\infty, -1/2)}$

4. Find the length of the curve WITHOUT a calculator and then use a calculator to check for A.) $x = \cos t,\ \ y = \sin t,\ \ 0 \le t \le 2\pi$ and B.) $x = \frac{1}{3}t^3,\ \ y = \frac{1}{2}t^2,\ \ 0 \le t \le 1$.

$x' = -\sin t \quad y' = \cos t$

$x' = t^2 \quad y' = t$

$\int_0^{2\pi} \sqrt{(-\sin t)^2 + (\cos t)^2}\, dt$

$\int_0^{2\pi} \sqrt{\sin^2 t + \cos^2 t}\, dt$

$\int_0^{2\pi} 1\, dt$

$t\Big|_0^{2\pi}$

$2\pi - 0 = \boxed{2\pi}$

$\int_0^1 \sqrt{(t^2)^2 + (t)^2}\, dt$

$\int_0^1 \sqrt{t^4 + t^2}\, dt$

$\int_0^1 \sqrt{t^2(t^2+1)}\, dt$

$\frac{1}{2}\int_0^1 2t\sqrt{t^2+1}\, dt$

$u = t^2 + 1$

$du = 2t\, dt$

$\frac{1}{2}\int_0^1 u^{1/2}$

$\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3} u^{3/2}\Big|_0^1$

$\frac{1}{3}(t^2+1)^{3/2}\Big|_0^1$

$\frac{1}{3}(2)^{3/2} - \frac{1}{3}(1)^{3/2}$

$\frac{2\sqrt{2}}{3} - \frac{1}{3} = \boxed{\frac{2\sqrt{2} - 1}{3}}$

$\frac{d^2y}{dx^2} = \dfrac{\cos t \cdot [\csc^3 t - \csc t \cot^2 t] - [-\cancel{\csc t} \cot t \cdot (-\cancel{\sin t})]}{[\cos t]^3} =$

5. A parmetric curve is defined by $x = \sin t$ and $y = \csc t$ for $0 \le t \le \frac{\pi}{2}$. This curve is

A.) ~~Increasing~~ and concave up.
B.) ~~Increasing~~ and concave down.
$\boxed{\text{C.)}}$ Decreasing and concave up
D.) Decreasing and concave down
E.) Decreasing with a point of inflection

$x' = \cos t \quad x'' = -\sin t$

$y' = -\csc t \cot t \quad y'' = -\csc t \cdot (-\csc^2 t) + \cot t(-\csc t \cot t)$

$y'' = \csc^3 t - \csc t \cot^2 t$

$\frac{dy}{dx} = \frac{\cos t}{-\csc t \cot t} = \frac{\cancel{\cos t}}{-\csc t \frac{\cancel{\cos t}}{\sin t}} = -\sin^2 t \ \boxed{\text{dec}}$

6. The parametric curve defined by $x = \ln t$, $y = t$ for $t > 0$ is identical to the graph of the function

A.) $y = \ln x$ for all real x.
B.) $y = \ln^x$ for $x > 0$.
$\boxed{\text{C.)}}$ $y = e^x$ for all real x.
D.) $y = e^x$ for $x > 0$
E.) $y = \ln(e^x)$ for $x > 0$

$x = \ln t$
$e \quad e$
$t = e^x$
$y = t$
$y = e^x$

7. Find the exact length of the curve WITHOUT a calculator and then use a calculator to check for A.) $y = \frac{1}{3}(x^2+2)^{\frac{3}{2}}$; on the interval $[0,3]$ B.) $y = x^{\frac{3}{2}}$; on the interval $[0,4]$

$\boxed{A}$ $y' = \frac{1}{\cancel{3}} \cdot \frac{\cancel{3}}{\cancel{2}}(x^2+2)^{1/2}(\cancel{2}x)$

$y' = x\sqrt{x^2+2}$

$\int_0^3 \sqrt{1 + [x^2(x^2+2)]}\,dx$

$\int_0^3 \sqrt{x^4+2x+1}\,dx$

$\int_0^3 \sqrt{(x^2+1)}\,dx$

$\int_0^3 x^2+1$

$\frac{x^3}{3} + x \Big|_0^3$

$\frac{27}{3} + 3 - 0 - 0$

$\boxed{6}$

$\boxed{B}$ $y' = \frac{3}{2}x^{1/2}$

$\frac{4}{9}\int_0^4 \frac{9}{4}\sqrt{1 + \frac{9}{4}x}$

$u = 1 + \frac{9}{4}x \quad u(0) = 1$
$du = \frac{9}{4}dx \quad u(4) = 10$

$\frac{4}{9}\int_0^4 u^{1/2}$

$\frac{4}{9} \cdot \frac{2}{3} u^{3/2} \Big|_1^{10}$

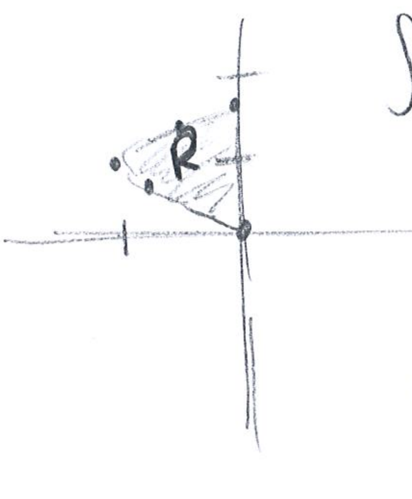
$\frac{8}{27}\left[10^{3/2} - 1^{3/2}\right]$

$\frac{8}{27} \cdot 10\sqrt{10} - 8/27$

$\boxed{\frac{80\sqrt{10}}{27} - \frac{8}{27}}$

8. Find the area enclosed by the curve $x = t^2 - 2t$, $y = \sqrt{t}$ and the y=axis.

t	$x = t^2 - 2t$	$y = \sqrt{t}$
0	0	0
1/2	-.75	.7
1	-1	1
1.75	-.4	1.3
2	0	1.4



$\int_0^2 \sqrt{t}\,\frac{d}{dt}[t^2 - 2t]\,dt$

$\int_0^2 \sqrt{t}(2t-2)\,dt$

$\int_0^2 2t^{3/2} - 2t^{1/2}\,dt$

$\boxed{.754247}$

8-12: Suppose a particle's x- and y-components' velocities are defined as $\frac{dx}{dt} = e^t - t$ and $\frac{dy}{dt} = e^t + t$. Answer the following.

9. Find the position vector $\vec{r}(t)$ given that $\vec{r}(0) = \langle 1, 1 \rangle$. $\boxed{P(t) = \langle e^t - \frac{1}{2}t^2, e^t + \frac{1}{2}t^2 \rangle}$

$\int e^t - t$

$e^t - \frac{1}{2}t^2 + C = x$

$e^0 - \frac{1}{2}(0)^2 + C = 1$

$C = 0$

$\int e^t + t$

$e^t + \frac{1}{2}t^2 + C = y$

$e^0 + \frac{1}{2}(0) + C = 1$

$C = 0$

10. Find the particle's coordinate position when $t = 3$.

$P(3) = \langle e^3 - \frac{9}{2}, e^3 + \frac{9}{2} \rangle$

11. Find the speed of the particle when $t = 2$.

$\text{speed} = \|v(t)\| = \sqrt{(e^2 - 2)^2 + (e^2 + 2)^2} = \boxed{10.826}$

12. Find the acceleration vector of the particle at any time t.

$a(t) = \langle e^t - 1, e^t + 1 \rangle$

13. Is the particle speeding up or slowing down when $t = 1.5$?

$v(1.5) = \langle 2.982, 5.982 \rangle$

$a(1.5) = \langle 3.482, 5.482 \rangle$

$DP = (2.982)(3.482) + (5.982)(5.482)$

$= \text{Positive}$

$\boxed{\therefore \text{ Speeding up}}$

14. Find the total distance traveled on the interval $[0,3]$.

$\int_0^3 \sqrt{(e^t - t)^2 + (e^t + t)^2}\, dt = \boxed{27.791}$

15. Find the displacement of the particle on the interval $[0,3]$

$P(3) = \langle e^3 - 9/2, e^3 + 9/2 \rangle$

$- P(0) = \langle 1, 1 \rangle$

$\langle e^3 - \frac{11}{2}, e^3 + \frac{7}{2} \rangle$

displacement =

$\sqrt{(e^3 - \frac{11}{2})^2 + (e^3 + 7/2)^2}$

$\boxed{27.731}$