AP CALCULUS (BC)
Unit 11
Vector Functions & Applications Worksheet

Name ____________
Date ____________
Period ____________

Suppose a particle's position is described by the vector $\overset{p(t)}{\cancel{\vec{r}(t)}} = \langle t^2 - 3, \ln(t+1) \rangle$. Answer the following.

1. State the domain of the vector function, $\vec{r}(t)$.

$\langle t^2 - 3, \ln(t+1) \rangle$

$\uparrow$ all Reals $\quad \uparrow$ $t+1>0$ $t>-1$

$d: (-1, \infty)$

2. Find the vector that represents the velocity of the particle at any time t.

$v(t) = \langle 2t, \frac{1}{t+1}(1) \rangle = \langle 2t, \frac{1}{t+1} \rangle$

3. How fast is the particle moving in the vertical direction when $t = 2$.

y - direction

$v(2) = \langle 2(2), \frac{1}{2+1} \rangle = \langle 4, \frac{1}{3} \rangle$

$\frac{1}{3}$ $\frac{\text{length unit}}{\text{time unit}}$

4. Find the vector that represents the acceleration of the particle at any time t.

$v(t) = \langle 2t, \frac{1}{t+1} \rangle = \langle 2t, (t+1)^{-1} \rangle$

$a(t) = \langle 2, \frac{-1}{(t+1)^2} \rangle$

5. Find a function $g(t)$ that represents the particle's speed at any time t.

$$\text{Speed} = \|v(t)\| = \sqrt{[v(x)]^2 + [v(y)]^2}$$
$$= \sqrt{(2t)^2 + \left(\frac{1}{t+1}\right)^2}$$
$$= \sqrt{4t^2 + \frac{1}{(t+1)^2}}$$

6. Find the speed of the particle when $t = 2$.

$$\text{Speed}(2) = \sqrt{4(2)^2 + \frac{1}{(2+1)^2}} = \sqrt{16 + \frac{1}{9}} = \sqrt{\frac{144+1}{9}} = \frac{\sqrt{145}}{3}$$

7. Is the particle speeding up or slowing down when $t = 2$?

$v(2) = \langle 4, 1/3 \rangle$

$a(2) = \langle 2, \frac{-1}{9} \rangle$

dot product

$4(2) + \frac{1}{3}\left(\frac{-1}{9}\right) = \text{positive}$

$\therefore$ speeding up

8. Find the total distance traveled by the particle on the interval $0 \le t \le 3$.

$$\int_0^3 \text{speed}\, dt = \int_0^3 \sqrt{4t^2 + \frac{1}{(t+1)^2}}\, dt = 9.3438892$$

9. Find the total displacement by the particle on the interval $0 \le t \le 3$.

$p(3) = \langle 6, \ln(4) \rangle$

$-\ p(0) = \langle -3, 0 \rangle$

$\langle 9, \ln(4) \rangle$

$\|p\| = \sqrt{(9)^2 + (\ln 4)^2} = 9.106$

P

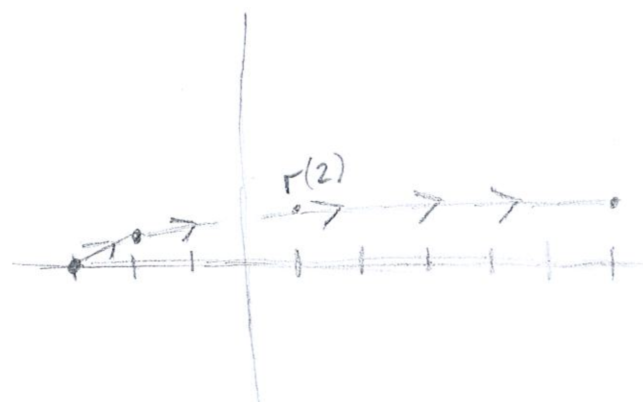
10. Sketch the path defined by $\vec{r}(t)$ on the interval $0 \le t \le 3$. Using appropriate initial points, sketch the $\vec{r}(2)$, $\vec{v}(2)$, and $\vec{a}(2)$ vectors on the path.

$p(0) = \langle -3, 0 \rangle$

$p(1) = \langle -2, \ln(2) \rangle = \langle -2, .7 \rangle$

$p(2) = \langle 1, \ln(3) \rangle = \langle 1, 1.1 \rangle$

$p(3) = \langle 6, \ln(4) \rangle = \langle 6, 1.4 \rangle$



Suppose a particle's velocity is described by the vector $\vec{v}(t) = \langle e^t, \sin t \rangle$. Answer the following.

11. Find the vector that represents the position of the particle at any time t given $\underset{p(0)}{\vec{r}(0)} = \langle 1, 0 \rangle$.

$\int v(t) = p(t)$ $\quad \int e^t = e^t + C \quad \int \sin t = -\cos t + C$

$p(0) = e^t + C \qquad p(0) = -\cos t + C$

$1 = e^0 + C \qquad 0 = -\cos(0) + C \qquad p(t) = \langle e^t, \cos t + 1 \rangle$

$C = 0 \qquad 0 = -1 + C$

$C = 1$

12. Find the vector that represents the acceleration of the particle at any time t.

$\frac{d}{dt}\, v(t) = \langle e^t, \sin t \rangle \frac{d}{dt}$

$a(t) = \langle e^t, \cos t \rangle$

13. Find the speed of the particle when $t = \pi/2$.

$\text{Speed} = \|v(t)\| = \sqrt{[e^t]^2 + [\sin t]^2}$

$\text{Speed} = \sqrt{e^{2t} + \sin^2 t}$

$\text{speed}(\frac{\pi}{2}) = \sqrt{e^{2(\frac{\pi}{2})} + (\sin \frac{\pi}{2})^2} = \sqrt{e^{\pi} + 1} \approx 4.9133$

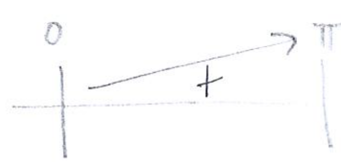
14. Find the total distance traveled on the interval $0 \le t \le \pi$.

$\int_0^{\pi} \sqrt{e^{2t} + \sin^2 t}\, dt = 22.3291$

15. Find where the maximum speed occurs on the interval $0 \le t \le \pi$.

$\frac{d}{dt}\text{Speed} = \left(\sqrt{e^{2t} + \sin^2 t}\right)\frac{d}{dt}$

$= \dfrac{e^{2t} + \sin t \cos t}{\sqrt{e^{2t} + \sin^2 t}}$



So at endpoint π is the max speed.

$p(t) = (t^{1/2}, t^2)$

Suppose a particle's position is described by the vector $\vec{r}(t) = \langle \sqrt{t}, t^2 \rangle$. Answer the following.

16. Find $\lim_{t \to 0^+} \vec{r}(t)$. $\langle \sqrt{0^+}, (0^2) \rangle = 0$

17. Find the vector that represents the velocity of $\vec{r}(t)$ at any time t.

$v(t) = \langle \frac{1}{2}t^{-1/2}, 2t \rangle = \langle \frac{1}{2\sqrt{t}}, 2t \rangle$

18. Find the vector that represents the acceleration of $\vec{r}(t)$ at any time t.

$a(t) = \langle -\frac{1}{4}t^{-3/2}, 2 \rangle = \langle \frac{-1}{4t^{3/2}}, 2 \rangle$

19. Evaluate $\vec{r}(1)$, $\vec{v}(1)$, and $\vec{a}(1)$.

$p(1) = \langle \sqrt{1}, 1^2 \rangle$ $\quad v(1) = \langle \frac{1}{2\sqrt{1}}, 2(1) \rangle$ $\quad \vec{a}(1) = \langle \frac{-1}{4(1)^{3/2}}, 2 \rangle$

$\vec{p}(1) = \langle 1, 1 \rangle$ $\quad \vec{v}(1) = \langle \frac{1}{2}, 2 \rangle$ $\quad = \langle -\frac{1}{4}, 2 \rangle$

20. Sketch a graph of $\vec{r}(t)$. In the same window, include the graphs of $\vec{r}(1)$, $\vec{v}(1)$, and $\vec{a}(1)$.

21. Find the speed of the particle when $t = 2$.

$\text{Speed} = \|v(t)\| = \sqrt{\left(\frac{1}{2\sqrt{t}}\right)^2 + (2t)^2} = \sqrt{\frac{1}{4t} + 4t^2} = \sqrt{\frac{1}{4(2)} + 4(2)^2}$

$= \sqrt{\frac{129}{8}} = \sqrt{16.125}$

≈ 4.016

22. Find the total distance traveled on the interval $0 \le t \le 2$.

$\int_0^2 \sqrt{\frac{1}{4t} + 4t^2} = 4.63125$

23. Find the total displacement on the interval $0 \le t \le 2$.

$P(2) - P(0)$

$\langle \sqrt{2}, 4 \rangle - \langle 0, 0 \rangle = \langle \sqrt{2}, 4 \rangle$ $\quad \|P\| = \sqrt{(\sqrt{2})^2 + 4^2} = 4.47214$

24. Describe the slope of the curve defined by $\vec{r}(t)$ when $t = 1$.

$r(t) = \langle \sqrt{t}, t^2 \rangle$ $\quad r'(t) = \langle \frac{1}{2\sqrt{t}}, 2t \rangle$ $\quad r'(1) = \langle \frac{1}{2}, 2 \rangle = \frac{2}{\frac{1}{2}} = 4$

Steep and positive

25. Evaluate $\int_0^2 \vec{r}(t)\,dt$. $\int_0^2 t^{1/2} + t^2 = 4.55228$

$u = t \quad v = \sin t$
$du = dt \quad dv = \cos t$

Suppose a particle's x- and y-components' velocities are defined as $\int \frac{dx}{dt} = \int \sin t$ and $\int \frac{dy}{dt} = \int t\cos t$. Answer the following.

$t\sin t - \int \sin t\, dt$
$t\sin t - -\cos t$

26. Find the position vector $\vec{r}(t)$ given that $\vec{p}(0) = \langle 2, -1 \rangle$.

$p(t) = \langle -\cos t + 3, \cos t + t\sin t - 2 \rangle$

$X = -\cos t + C$
$2 = -\cos(0) + C$
$2 = -1 + C$
$C = 3$

$y = t\sin t + \cos t + C$
$-1 = 0\sin(0) + \cos(0) + C$
$-1 = 1 + C$
$C = -2$

27. Find the particle's coordinate position when $t = 1$.

$p(1) = \langle 2.4597, -.6182 \rangle$

28. Find the speed of the particle when $t = 2$.

Speed $= \sqrt{(\sin t)^2 + (t\cos t)^2} = 1.23269$

29. Find the acceleration vector of the particle at any time t.

$\vec{v}(t) = \langle \sin t, t\cos t \rangle$
$\vec{a}(t) = \langle \cos t, t(-\sin t) + \cos t(1) \rangle = \langle \cos t, \cos t - t\sin t \rangle$

30. Is the particle speeding up or slowing down when $t = 1.5$?

$\vec{v}(1.5) = \langle .9975, .1061 \rangle$
$\vec{a}(1.5) = \langle .0707, -1.42551 \rangle$

dot product
$= .0705 + -.151247 =$ neg
so decreasing

31. Is there a time t when the particle is not moving?

$\frac{dx}{dt} = 0 = \sin t$
$t = 0, \pi, 2\pi, \ldots$

$\frac{dy}{dx} = 0 = t\cos t = 0$
$t = 0 \quad \cos t = 0$
$t = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

Only at $t = 0$ that is the only time x & y have $v(t) = 0$.

32. Is there a time t when the particle is not accelerating?