AP Calculus BC $\text{Arc Length}: \int_a^b \sqrt{1+[f'(x)]^2}\,dx$ Name______________________Pd.____

Supplement Arc Length: Rectangular and Parametric P, P, & V Day 5

1-4: Set up the integral that represents the length of the curve.

1. $y = 3x+1$, $[0,3]$

$f'(x) = 3$

$\text{Arc length} = \int_0^3 \sqrt{1+(3)^2}\,dx$

$= \int_0^3 \sqrt{10}\,dx$

$= \sqrt{10}x\Big|_0^3 = 3\sqrt{10} - \cancel{0\sqrt{10}}$

$= \boxed{3\sqrt{10}}$

2. $y = 9-3x$, $[1,3]$

$f'(x) = -3$

$\text{Arc length} = \int_1^3 \sqrt{1+(-3)^2}\,dx$

$\int_1^3 \sqrt{10}\,dx$

$\sqrt{10}x\Big|_1^3 = 3\sqrt{10} - 1\sqrt{10}$

$\boxed{2\sqrt{10}}$

3. $y = x^{\frac{3}{2}}$, $[1,2]$

$f'(x) = \frac{3}{2}x^{1/2}$

$\text{Arclength} = \int_1^2 \sqrt{1+\left(\frac{3}{2}x^{1/2}\right)^2}\,dx$

$\frac{4}{9}\int_1^2 \frac{9}{4}\sqrt{1+\frac{9}{4}x}$

$u = 1+\frac{9}{4}x$

$du = \frac{9}{4}dx$

$\frac{4}{9}\int u^{1/2}\,du$

$u(1) = 1+\frac{9}{4}(1) = \frac{13}{4}$

$\frac{4}{9}\frac{2}{3}u^{3/2}\Big|_{\frac{13}{4}}^{\frac{11}{2}}$

$u(2) = 1+\frac{9}{4}(2) =$

$\frac{2}{2}+\frac{9}{2} = \frac{11}{2}$

$\frac{8}{27}\left(\frac{11}{2}\right)^{3/2} - \frac{8}{27}\left(\frac{13}{4}\right)^{3/2}$

$\approx \boxed{2.086}$

4. $y = \ln(\cos x)$, $0 \le y \le \frac{\pi}{4}$

$y' = \frac{1}{\cos x} \cdot -\sin x = -\tan x$

$\text{Arclength} = \int_0^{\pi/4} \sqrt{1+(-\tan x)^2}\,dx$

$= \int_0^{\pi/4} \sqrt{1+\tan^2 x}\,dx$

Identity $1+\tan^2 x = \sec^2 x$

$\int_0^{\pi/4} \sqrt{\sec^2 x}\,dx$

$\int \sec x \Big|_0^{\pi/4}$

$\ln|\sec x + \tan x| \Big]_0^{\pi/4}$

$\ln|\sec\frac{\pi}{4} + \tan\frac{\pi}{4}| - \ln|\sec(0) + \tan(0)|$

$\ln|\frac{2}{\sqrt{2}}+1| - \cancel{\ln|1+0|}^{\,0}$

$\boxed{\ln|\frac{2}{\sqrt{2}}+1|}$

Arc length $= \int_{t_0}^{t_1} \sqrt{[x'(t)]^2+[y'(t)]^2}\,dt$

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5-8: Set up the integral that represents the length of the curve.

5. $C(t)=(3t+1, 9-4t),\quad 0 \le t \le 2$

$x'(t)=3 \quad y'(t)=-4$

$\int_0^2 \sqrt{(3)^2+(-4)^2}\,dt$

$\int_0^2 \sqrt{25}\,dt$

$5t\Big|_0^2$

$5(2)-5(0)$

$\boxed{10}$

6. $x=1+2t,\quad y=2+4t,\quad 1 \le t \le 4$

$x'(t)=2 \quad y'(t)=4$

$\int_1^4 \sqrt{(2)^2+(4)^2}\,dt$

$\int_1^4 \sqrt{20}\,dt = \int_1^4 2\sqrt{5}\,dt$

$2\sqrt{5}\,t\Big|_1^4$

$8\sqrt{5}-2\sqrt{5}$

$\boxed{6\sqrt{5}}$

7. $x=2t^2,\quad y=3t^2-1,\quad 0 \le t \le 4$

$x'(t)=4t \quad y'(t)=6t$

$\int_0^4 \sqrt{(4t)^2+(6t)^2}\,dt$

$\int_0^4 \sqrt{16t^2+36t^2}\,dt$

$\int_0^4 \sqrt{52t^2}\,dt$

$\int_0^4 2\sqrt{13}\,t\,dt$

$\frac{\cancel{2}\sqrt{13}\,t^2}{\cancel{2}}\Big|_0^4$

$16\sqrt{13}-0\sqrt{13}$

$\boxed{16\sqrt{13}}$

8. $C(x)=\left(t^3+1, t^2-3\right)\ [0,1]$

$x'(t)=3t^2 \quad y'(t)=2t$

$\int_0^1 \sqrt{(3t^2)^2+(2t)^2}\,dt$

$\int_0^1 \sqrt{9t^4+4t^2}\,dt$

$\int_0^1 \sqrt{t^2(9t^2+4)}\,dt$

$\frac{1}{18}\int_0^1 18t\sqrt{9t^2+4}\,dt$

$u=9t^2+4$

$du=18t\,dt$

$u(0)=4$

$u(1)=13$

$\frac{1}{18}\int_4^{13} u^{1/2}\,du$

$\frac{1}{18}\cdot\frac{2}{3}u^{3/2}\Big|_4^{13}$

$\frac{1}{27}(13)^{3/2}-\frac{1}{27}(4)^{3/2} \approx \boxed{1.44}$