

Arc Length: Rectangular and Parametric

P, P, & V Day 5

Make sure you can plug these into your calculator!!!

1-4: Set up the integral that represents the length of the curve. Then use your calculator to find the length correct to four decimal places.

1. $y = \sin x, 0 \leq x \leq \pi$

$$f'(x) = \cos x$$

$$\int_0^{\pi} \sqrt{1 + \cos^2 x} \, dx = \boxed{3.8202}$$

2. $y = xe^{-x}, 0 \leq x \leq 2$

$$f'(x) = xe^{-x}(-1) + e^{-x}(1) = e^{-x}(1-x)$$

$$\int_0^2 \sqrt{1 + [e^{-x}(1-x)]^2} \, dx$$

$$\int_0^2 \sqrt{1 + e^{-2x}(1-x)^2} \, dx = \boxed{2.1024}$$

2nd 1 $\int((1 + (\cos(x))^{1/2})^{1/2}, x, 0, \pi)$

3. $x = \sqrt{y-y}, 1 \leq y \leq 4$

$$x'(y) = \frac{1}{2}y^{-1/2} - 1$$

$$\int_1^4 \sqrt{1 + \left(\frac{1}{2\sqrt{y}} - 1\right)^2} \, dy \approx \boxed{3.6095}$$

4. $x = y^2 - 2y, 0 \leq y \leq 2$

$$x'(y) = 2y - 2$$

$$\int_0^2 \sqrt{1 + (2y-2)^2} \, dy = \boxed{2.9579}$$

5-8: Set up the integral that represents the length of the curve. Then use your calculator to find the length correct to four decimal places.

5. $x = t + e^{-t}, y = t - e^{-t}, 0 \leq t \leq 2$

$$x'(t) = 1 + e^{-t}(-1) \quad y'(t) = 1 - e^{-t}(-1)$$

$$\int_0^2 \sqrt{(1 - e^{-t})^2 + (1 + e^{-t})^2} \, dt \approx \boxed{3.1416}$$

6. $x = t^2 - t, y = t^4, 1 \leq t \leq 4$

$$x'(t) = 2t - 1 \quad y'(t) = 4t^3$$

$$\int_1^4 \sqrt{(2t-1)^2 + 16t^6} \, dt = \boxed{255.3756}$$

7. $x = t - 2\sin t, y = 1 - 2\cos t, 0 \leq t \leq 4\pi$

$$x'(t) = 1 - 2\cos t \quad y'(t) = -2(-\sin t) = 2\sin t$$

$$\int_0^{4\pi} \sqrt{(1 - 2\cos t)^2 + (2\sin t)^2} \, dt \approx \boxed{26.7298}$$

8. $x = t + \sqrt{t}, y = t - \sqrt{t}, 0 \leq t \leq 1$

$$x'(t) = 1 + \frac{1}{2}t^{-1/2} \quad y'(t) = 1 - \frac{1}{2}t^{-1/2}$$

$$\int_0^1 \sqrt{\left(1 + \frac{1}{2\sqrt{t}}\right)^2 + \left(1 - \frac{1}{2\sqrt{t}}\right)^2} \, dt = 2.0915$$