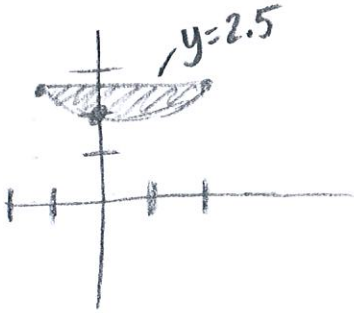
$\int_{t_0}^{t_1} y(t)\frac{d}{dt}x(t)\,dt$

AP Calculus BC Name_______________________

Supplement: Parametric Equations: Finding Area P, P, & V Day 3

1. Find the area enclosed by the curve $c(t)=\left(t-\frac{1}{t},t+\frac{1}{t}\right)$ and the line $y=2.5$.

t	x	y
0	UD	UD
.5	-1.5	2.5
1	0	2
1.5	.8	2.2
2	1.5	2.5



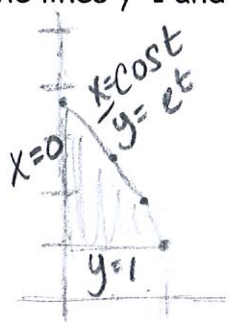
$\int_{.5}^{2}\left(2.5-\left(t+\frac{1}{t}\right)\right)\frac{d}{dt}\left[t-\frac{1}{t}\right]$

$\int_{.5}^{2}(2.5-t-t^{-1})(1+t^{-2})\,dt$

$\boxed{\frac{15}{4}-4\ln(2)}$

2. Consider the curve $r(t)=(\cos t, e^t)$ on the interval $0\le t\le\frac{\pi}{2}$. Find the area of the region bounded by r(t) and the lines y=1 and x=0.

t	cos t	e^t
0	1	1
π/6	.9	1.7
π/4	.7	2.2
π/3	.5	2.8
π/2	0	4.8



$\int_0^{\pi/2}(e^t-1)\frac{d}{dt}[\cos t]\,dt$

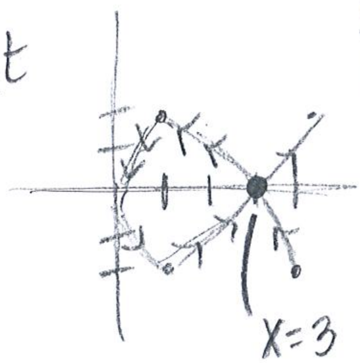
$\int_0^{\pi/2}-\sin t(e^t-1)\,dt$

$=\boxed{\frac{e^{\pi/2}-1}{2}}$

3. Find the area of the region enclosed by the loop of the curve with parametric equations $x=t^2, y=t^3-3t$.

$3=x$
$3=t^2$
$t=\pm\sqrt{3}$

t	t^2	t^3-3t
-2	4	-2
-1	1	2
0	0	0
1	1	-2
2	4	2



$\int_{-\sqrt{3}}^{\sqrt{3}}(t^3-3t)\frac{d}{dt}(t^2)\,dt$

$\int_{-\sqrt{3}}^{\sqrt{3}}2t(t^3-3t)\,dt$

$\left|-\frac{24\sqrt{3}}{5}\right|=\boxed{\frac{24\sqrt{3}}{5}}$

4. The parametric form for a cardioid is $\begin{cases}x=2a\cos t-a\cos 2t\\ y=2a\sin t-a\sin 2t\end{cases}$, where a is an arbitrary constant. Find the area of the region enclosed by the cardiod.

$\int_0^{2\pi}(2a\sin t-a\sin 2t)\frac{d}{dt}[2a\cos t-a\cos 2t]\,dt$

$\int_0^{2\pi}(2a\sin t-a\sin 2t)(-2a\sin t+2a\cos 2t)\,dt$

$=\boxed{6\pi a^2}$