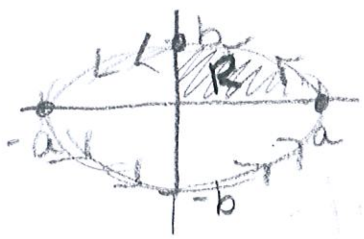
AP Calculus BC $\int_{t_1}^{t_2} y(t)x'(t)\,dt$ Name______________________________

Parametric Equations: Finding Area P, P, & V Day 3

1. Use the parametric equations of an ellipse $x = a\cos\theta$, $y = b\sin\theta$, $0 \le \theta \le 2\pi$, to find the area that it encloses.

t	$a\cos\theta$	$b\sin\theta$
0	a	0
$\pi/2$	0	b
π	-a	0
$\frac{3\pi}{2}$	0	-b
2π	a	0



$4\int_0^{\frac{\pi}{2}} (b\sin\theta)\frac{d}{d\theta}(a\cos\theta)\,d\theta$

$\int_0^{\frac{\pi}{2}} (b\sin\theta)(-a\sin\theta)\,d\theta$

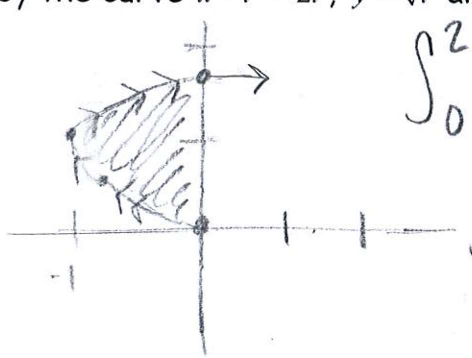
$\left|-4ab\int_0^{\pi/2} \sin^2\theta\right|$

$4ab\left(\frac{\pi}{4}\right) = \boxed{\pi ab}$

---

2. Find the area enclosed by the curve $x = t^2 - 2t$, $y = \sqrt{t}$ and the y=axis.

t	t^2-2t	$\sqrt{t}$
0	0	0
.5	-.75	.7
1	-1	1
1.75	-.4	1.3
2	0	1.4



$\int_0^2 [\sqrt{t}]\frac{d}{dt}[t^2-2t]\,dt$

$\int_0^2 [\sqrt{t}](2t-2)\,dt$

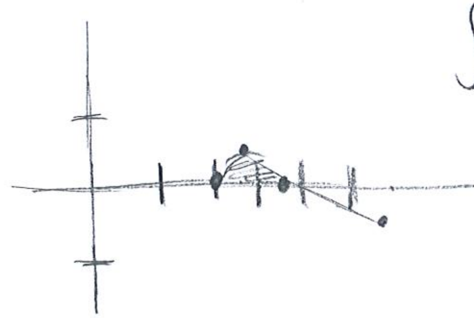
$\int_0^2 2t^{3/2} - 2t^{1/2}\,dt$

$\boxed{.754247}$

---

3. Find the area enclosed by the x-axis and the curve $x = 1 + e^t$, $y = t - t^2$.

t	$1+e^t$	$t-t^2$
0	2	0
.5	2.6	.25
1	3.7	0
1.5	5.4	-.75



$\int_0^1 (t-t^2)\frac{d}{dt}(1+e^t)$

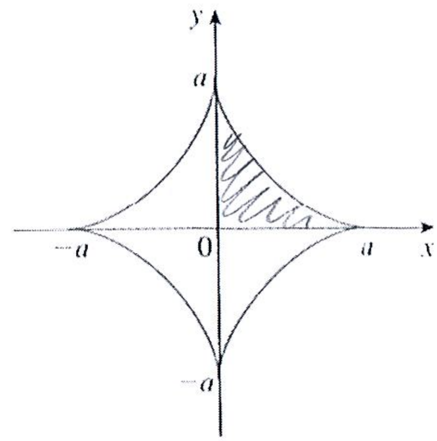
$\int_0^1 (t-t^2)(e^t)\,dt$

$= \boxed{.281718}$

AP Calculus BC Name_______________________

Parametric Equations: Finding Area P, P, & V Day 3

4. Find the area of the region enclosed by the asteroid $x = a\cos^3\theta$, $y = a\sin^3\theta$.



$$4\int_0^{\frac{\pi}{2}} a\sin^3\theta \frac{d}{d\theta}\left[a\cos^3\theta\right] d\theta$$

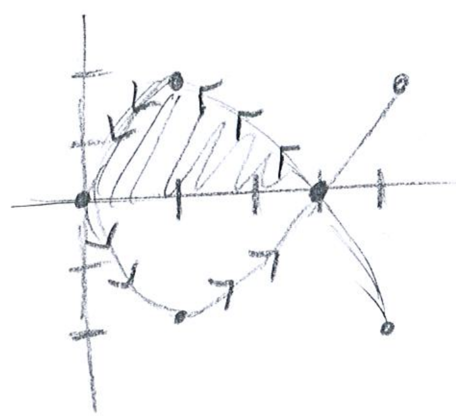
$$4\int_0^{\frac{\pi}{2}} a\sin^3\theta \left[3a\cos^2\theta(-\sin\theta)\right] d\theta$$

$$4\int_0^{\frac{\pi}{2}} -3a^2\sin^4\theta\cos^2\theta \, d\theta$$

$$4\left[\frac{3a^2\pi}{32}\right] = \boxed{\frac{3a^2\pi}{8}}$$

5. Let $\Re$ be the region enclosed in the loop of the curve in $x = t^2$, $y = t^3 - 3t$.
Find the area of $\Re$.

t	t^2	t^3-3t
-2	4	-2
-1	1	2
0	0	0
1	1	-2
2	4	2



When does $x=3$?

$3 = t^2$

$t = \sqrt{3}, -\sqrt{3}$

$$2\int_0^{\sqrt{3}} (t^3-3t)\frac{d}{dt}(t^2)\, dt$$

$$\int_0^{\sqrt{3}} 2t(t^3-3t)\, dt$$

$$\int_0^{\sqrt{3}} 2t^3-6t^2\, dt = 2\left(\frac{12\sqrt{3}}{5}\right) = \boxed{\frac{24\sqrt{3}}{5}}$$