AP Calculus BC Name______________________________

Parametric Equations: Finding Derivatives P, P, & V Day 2

1-2: Find $\frac{dy}{dx}$.

1. $x = t\sin t, \quad y = t^2 + t$

$\frac{dx}{dt} = t\cos t + \sin t(1)$ $\quad \frac{dy}{dt} = 2t+1$

$\frac{dy}{dx} = \frac{2t+1}{t\cos t + \sin t}$

2. $x = \frac{1}{t}, \quad y = \sqrt{t}e^{-t}$

$x = \frac{1}{t} = t^{-1}$

$\frac{dx}{dt} = -1t^{-2} = \frac{-1}{t^2}$

$\frac{dy}{dt} = t^{1/2}e^{-t}(-1) + e^{-t}\frac{1}{2}t^{-1/2}$

$= \frac{\cancel{2t}-\sqrt{t}e^{-t}}{2\sqrt{t}} + \frac{e^{-t}}{2\sqrt{t}} = \frac{-2te^{-t}+e^{-t}}{2\sqrt{t}}$

$\frac{dy}{dx} = \frac{\frac{-2te^{-t}+e^{-t}}{2\sqrt{t}}}{\frac{-1}{t^2}} \cdot \frac{\frac{t^2}{-1}}{\frac{t^2}{-1}} = \frac{2t^3e^{-t} - e^{-t}t^2}{2\sqrt{t}}$

$= \frac{1}{2}t^{5/2}e^{-t} - t^{3/2}e^{-t}$

3-6: Find an equation of the tangent to the curve at the point corresponding to the given value for the parameter.

3. $x = 1 + 4t - t^2, \quad y = 2 - t^3; \quad t = 1$

$\frac{dx}{dt} = 4 - 2t \quad \frac{dy}{dt} = -3t^2$

$x(1) = 1+4-1 = 4$

$y(1) = 2-(1)^3 = 1$

$(4,1)$

$\left.\frac{dy}{dx}\right|_{t=1} = \frac{-3(1)^2}{4-2(1)} = \frac{-3}{2}$

$\boxed{y - 1 = -\frac{3}{2}(x-4)}$

4. $x = t - t^{-1}, \quad y = 1 + t^2; \quad t = 1$

$x(1) = 1 - \frac{1}{1} = 0$

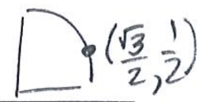
$y(1) = 1 + 1^2 = 2$

$\frac{dx}{dt} = 1 + 1t^{-2} \quad \frac{dy}{dt} = 2t$

$(0,2)$

$\left.\frac{dy}{dx}\right|_{t=1} = \frac{2(1)}{1+\frac{1}{(1)^2}} = \frac{2}{2} = 1$

$\boxed{y - 2 = x}$



5. $x = t\cos t, \quad y = t\sin t; \quad t = \pi$

$\frac{dx}{dt} = t(-\sin t) + \cos t(1) \quad \frac{dy}{dt} = t\cos t + \sin t(1)$

$\frac{dy}{dx} = \frac{\pi\cos\pi + \sin\pi}{-\pi\sin\pi + \cos\pi} = \frac{\pi(-1)+0}{-\pi(0)-1} = \frac{-\pi}{-1} = \pi$

$x(\pi) = \pi\cos\pi = -\pi$

$y(\pi) = \pi\sin\pi = 0$

$(-\pi, 0)$

$\boxed{y = \pi(x+\pi)}$

6. $x = \sin^3\theta, \quad y = \cos^3\theta; \quad \theta = \frac{\pi}{6}$

$\frac{dx}{d\theta} = 3\sin^2\theta\cos\theta \quad \frac{dy}{d\theta} = 3\cos^2\theta(-\sin\theta)$

$\frac{dy}{dx} = \frac{-\cancel{3}(\cos\frac{\pi}{6})^{\cancel{2}}\cancel{\sin\frac{\pi}{6}}}{\cancel{3}(\sin\frac{\pi}{6})^{\cancel{2}}\cancel{\cos\frac{\pi}{6}}} = -\frac{\frac{\sqrt{3}}{2}/\frac{\cancel{2}}{1}}{\frac{1}{\cancel{2}}/\frac{2}{1}} = -\sqrt{3}$

$x(\frac{\pi}{6}) = (\sin\frac{\pi}{6})^3 = (\frac{1}{2})^3 = \frac{1}{8}$

$y(\frac{\pi}{6}) = (\cos\frac{\pi}{6}) = (\sqrt{3}/2)^3 = 3\sqrt{3}/8$

$\boxed{y - \frac{3\sqrt{3}}{8} = -\sqrt{3}(x - \frac{1}{8})}$

7-8: Find an equation fo the tangent to the curve at the given point by two methods: A.) With out eliminating the parameter and B.) by first eliminating the parameter.

A

7. $x = 1 + \ln t, \quad y = t^2 + 2; \quad (1,3)$

$y = t^2 + 2$

$3 = t^2 + 2$

-2

$t^2 = 1 \quad \boxed{t=1}$

$\frac{dx}{dt} = \frac{1}{t} \quad \frac{dy}{dt} = 2t$

$\left.\frac{dy}{dx}\right|_{t=1} = \frac{2(1)}{1/1} = 2$

$\boxed{y - 3 = 2(x-1)}$

B

$x = 1 + \ln t$

$\ln t = x - 1$

$t = e^{x-1}$

$y = (e^{x-1})^2 + 2 = e^{2x-2} + 2$

$\frac{dy}{dx} = e^{2x-2}(2)$

$\left.\frac{dy}{dx}\right|_{x=1} = e^0(2) = 2$

$\boxed{y - 3 = 2(x-1)}$

A

8. $x = 1 + \sqrt{t}, \quad y = e^{t^2}; \quad (2, e)$

$x = 1 + \sqrt{t}$

$2 = 1 + \sqrt{t}$

$1 = \sqrt{t}$

$t = 1$

$\frac{dx}{dt} = \frac{1}{2}t^{-1/2} \quad \frac{dy}{dt} = e^{t^2}(2t)$

$\left.\frac{dy}{dx}\right|_{t=1} = \frac{e^1(2)(1)}{1/2\sqrt{1}} = \frac{2e}{1/2} = 4e$

$\boxed{y - e = 4e(x-2)}$

B

$x = 1 + \sqrt{t}$

$\sqrt{t} = x - 1$

$t = (x-1)^2$

$y = e^{[(x-1)^2]^2} = e^{(x-1)^4}$

$\frac{dy}{dx} = e^{(x-1)^4} \cdot 4(x-1)^3(1)$

$\boxed{y - e = 4e(x-2)}$

$dy/dx|_{x=2} = e \cdot 4(1)^3(1) = 4e$

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Parametric Equations: Finding Derivatives P, P, & V Day 2

9-10: Find an equation of the tangent(s) to the curve at the given point.

9. $x = 6\sin t, \quad y = t^2 + t; \quad (0,0)$

$y = t^2 + t$

$0 = t^2 + t$

$t = 0$

$\frac{dx}{dt} = 6\cos t \quad \frac{dy}{dt} = 2t + 1$

$\left.\frac{dy}{dx}\right|_{t=0} = \frac{2(0)+1}{6\cos(0)} = \frac{1}{6}$

$\boxed{y - 0 = \frac{1}{6}(x - 0)}$

10. $x = \cos t + \cos 2t, \quad y = \sin t + \sin 2t; \quad (-1,1)$ & $t = \frac{\pi}{2}$

$\frac{dy}{dx} = \frac{\cos t + \cos(2t)(2)}{-\sin t - \sin(2t)(2)}$

$\frac{dy}{dx} = \frac{\cos\frac{\pi}{2} + \cos(\frac{2\pi}{2})(2)}{-\sin\frac{\pi}{2} - \sin(\frac{2\pi}{2})(2)} = \frac{-1(2)}{-(1)} = 2$

$\boxed{y - 1 = 2(x + 1)}$

11-16: Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. For which values of t is the curve concave upward?

11. $x = t^2 + 1, \quad y = t^2 + t$

$x'(t) = 2t \quad x''(t) = 2$

$y'(t) = 2t + 1 \quad y''(t) = 2$

$\frac{dy}{dx} = \boxed{\frac{2t+1}{2t}}$

$\frac{d^2y}{dx^2} = \frac{2t(2) - (2t+1)(2)}{(2t)^3} = \frac{\cancel{4t} - \cancel{4t} - 2}{8t^3} = \boxed{\frac{-1}{4t^3}}$

neg / neg | + (0)

CC↑ $\frac{-1}{4t^3} > 0$ if $\boxed{t < 0}$ CC↑

12. $x = t^3 + 1, \quad y = t^2 - t$

$x'(t) = 3t^2 \quad x''(t) = 6t$

$y'(t) = 2t - 1 \quad y''(t) = 2$

$\frac{dy}{dx} = \frac{2t - 1}{3t^2}$

CC↑ $\boxed{0 < t < 1}$

$\frac{d^2y}{dx^2} = \frac{3t^2(2) - (2t-1)6t}{[3t^2]^3} = \frac{6t^2 - 12t^2 + 6t}{27t^6}$

$\frac{d^2y}{dx^2} = \frac{-6t^2 + 6t}{27t^6} = \frac{6t(-t+1)}{27t^6} = \frac{2(-t+1)}{9t^5}$ − 0 + 1 − pos

13. $x = e^t, \quad y = te^{-t}$

$x'(t) = e^t \quad x''(t) = e^t$

$y'(t) = e^{-t}(1-t) \quad y''(t) = e^{-t}(-2+t)$

$y' = te^{-t}(-1) + e^{-t}(1)$

$y'' = e^{-t}(-1) + (1-t)e^{-t}(-1)$

$e^{-t}(-1 - 1 + t)$

$\frac{dy}{dx} = \frac{e^{-t}(1-t)}{e^t} = \boxed{\frac{1-t}{e^{2t}}}$

$\frac{d^2y}{dx^2} = \frac{\cancel{e^t}e^{-t}(-2+t) - \cancel{e^{-t}}(1-t)(\cancel{e^t})}{e^{3t}} = \frac{-2+t-1+t}{e^{3t}}$

$\frac{d^2y}{dx^2} = \boxed{\frac{2t-3}{e^{3t}}}$ − 1.5 + Always +

$\boxed{CC\uparrow \ t > 3/2}$

14. $x = t^2 + 1, \quad y = e^t - 1$

$x'(t) = 2t \quad x''(t) = 2$

$y'(t) = e^t \quad y''(t) = e^t$

$\frac{dy}{dx} = \boxed{\frac{e^t}{2t}}$

$\frac{d^2y}{dx^2} = \frac{2te^t - e^t(2)}{(2t)^3} = \boxed{\frac{2e^t(t-1)}{8t^3}}$ − 1 + / − 0 +

$\boxed{CC\uparrow \text{ when } t < 0 \text{ or } t > 1}$

15. $x = 2\sin t, \quad y = 3\cos t, \quad 0 < t < 2\pi$

$\boxed{CC\uparrow \ \frac{\pi}{2} < t < \frac{3\pi}{2}}$

$x'(t) = 2\cos t \quad x''(t) = -2\sin t$

$y' = -3\sin t \quad y'' = -3\cos t$

$\frac{dy}{dx} = \frac{-3\sin t}{2\cos t} = \boxed{\frac{-3}{2}\tan t}$

$\frac{d^2y}{dx^2} = \frac{2\cos t(-3\cos t) - (-3\sin t)(-2\sin t)}{[2\cos t]^3}$

$\frac{d^2y}{dx^2} = \frac{-6\cos^2 t - 6\sin^2 t}{8\cos^3 t} = \frac{-6(1)}{8\cos^3 t} = \boxed{\frac{-3}{4\cos^3 t}}$

neg 0 + $\frac{\pi}{2}$ − $\frac{3\pi}{2}$ +

16. $x = \cos 2t, \quad y = \cos t, \quad 0 < t < \pi$

$x'(t) = -2\sin 2t \quad x''(t) = -4\cos 2t$

$y' = -\sin t \quad y'' = -\cos t$

$\frac{dy}{dx} = \boxed{\frac{-\sin t}{-2\sin 2t}}$

$\frac{d^2y}{dx^2} = \frac{(-2\sin 2t)(-\cos t) - (-\sin t)(-4\cos 2t)}{[-2\sin(2t)]^3}$

$\frac{d^2y}{dx^2} = \boxed{\frac{2\sin 2t\cos t - 4\sin t\cos(2t)}{-8\sin(2t)}}$

$\boxed{\text{don't worry about CC}}$

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Parametric Equations: Finding Derivatives — P, P, & V Day 2

17-20: Find the points on the curve where the tangent is horizontal or vertical. If you have a graphing device, graph the curve to check your work.

17. $x = t^3 - 3t, \quad y = t^2 - 3$

$\frac{dx}{dt} = 3t^2 - 3 \qquad \frac{dy}{dt} = 2t$

Horizontal $\frac{dy}{dt} = 0$

$2t = 0$

$t = 0$

$\boxed{(0,-3)}$

Vertical $\frac{dx}{dt} = 0$

$3t^2 - 3 = 0$

$3t^2 = 3$

$t^2 = 1$

$t = \pm 1$

$t = 1 \qquad t = -1$

$(1-3, 1-3) \qquad (-1+3, 1-3)$

$\boxed{(-2,-2) \quad (2,-2)}$

18. $x = t^3 - 3t, \quad y = t^3 - 3t^2$

$\frac{dx}{dt} = 3t^2 - 3 \qquad \frac{dy}{dt} = 3t^2 - 6t$

Horizontal $\frac{dy}{dt} = 0$

$3t^2 - 6t = 0$

$3t(t-2)$

$t = 0 \quad t = 2$

$\boxed{(0,0) \quad (2,-4)}$

$8 - 3(2)$

$8 - 3(4)$

Vertical $\frac{dx}{dt} = 0$

$3t^2 - 3 = 0$

$3t^2 = 3$

$t^2 = 1$

$t = \pm 1$

$\boxed{(-2,-2) \quad (2,-4)}$

$1-3 \qquad -1+3$

$+1-3(1) \qquad -1-3$

19. $x = \cos\theta, \quad y = \cos(3\theta)$

$\frac{dx}{dt} = -\sin\theta \qquad \frac{dy}{dt} = -3\sin(3\theta)$

Horizontal $\frac{dy}{dt} = 0$

$-3\sin(3\theta) = 0$

$\sin 3\theta = 0$

$\theta = 0, \pi, 2\pi, \ldots$

$3\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \ldots \pi$

$\boxed{(1,1) \quad (\tfrac{1}{2},-1) \quad (-\tfrac{1}{2},1)} \quad (-1,-1)$

Vertical $\frac{dx}{dt} = 0$

$-\sin\theta = 0$

$\sin\theta = 0$

$\theta = 0, \pi, 2\pi, \ldots$

$\theta = 0$

$\cancel{\cos 0, \cos(0)}$

$\cancel{\boxed{(1,1)}}$

$\cancel{\cos\pi, \cos 3\pi}$

$\cancel{\boxed{(-1,-1)}}$

Aren't vertical. Only horiz.

Can't be both :)

20. $x = e^{\sin\theta}, \quad y = e^{\cos\theta}$

$\frac{dx}{dt} = e^{\sin\theta}\cos\theta \qquad \frac{dy}{dt} = e^{\cos\theta}(-\sin\theta)$

omit :)

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Parametric Equations: Finding Derivatives P, P, & V Day 2

21. Show that the curve $x = \cos t$, $y = \sin t \cos t$ has two tangents at $(0,0)$ and find their equations. Sketch the curve.

$X = \cos t$
$x = 0$
$\cos t = 0$
$t = \frac{\pi}{2}, \frac{3\pi}{2}$

Tangents

$\boxed{1.}$ Point $(0,0)$

$\boxed{2}$ Slope

$t = \frac{\pi}{2}$ $\quad t = \frac{3\pi}{2}$

$m = 1$ $\quad m = -1$

$\boxed{\begin{array}{l} y = x \\ y = -x \end{array}}$

$$\frac{dy}{dx} = \frac{\sin t(-\sin t) + \cos t \cos t}{-\sin t}$$

$$\frac{dy}{dx} = \frac{-\sin^2 t + \cos^2 t}{-\sin t}$$

$$\left.\frac{dy}{dx}\right|_{t=\frac{\pi}{2}} = \frac{-(\sin\frac{\pi}{2})^2 + \cos\frac{\pi}{2}}{-\sin\frac{\pi}{2}} \qquad \frac{-1+0}{-1} = 1$$

$$\left.\frac{dy}{dx}\right|_{t=\frac{3\pi}{2}} = \frac{-(\sin\frac{3\pi}{2})^2 + \cos\frac{3\pi}{2}}{-\sin\frac{3\pi}{2}} \qquad \frac{-1+0}{1} = -1$$

22. At what points on the curve $x = 2t^3$, $y = 1 + 4t - t^2$ does the tangent line have a slope 1?

$$\frac{dy}{dx} = \frac{4-2t}{6t^2} = 1$$

$4 - 2t = 6t^2$

$6t^2 + 2t - 4 = 0$

$3t^2 + t - 2 = 0$

$(3t-2)(t+1) = 0$

$t = \frac{2}{3} \quad t = -1$

$t = \frac{2}{3}$ $\boxed{\left(\frac{16}{27}, \frac{29}{9}\right)}$ $\quad t = -1$ $\boxed{(-2, -4)}$

$X = 2\left(\frac{2}{3}\right)^3 = 2\left(\frac{8}{27}\right) = \frac{16}{27}$

$y = 1 + 4\left(\frac{2}{3}\right) - \left(\frac{2}{3}\right)^2$

$y = 1 + \frac{8}{3} - \frac{4}{9}$

$y = \frac{9}{9} + \frac{24}{9} - \frac{4}{9} = \frac{29}{9}$

$X = 2(-1)^3 = -2$

$y = 1 + 4(-1) - (-1)^2 = -4$

23. Find the equations of the tangents to the curve $x = 3t^2 + 1$, $y = 2t^3 + 1$ that pass through the point (4,3).

$X = 3t^2 + 1$

$4 = 3t^2 + 1$

$3t^2 = 3$

$t^2 = 1$

$t = \pm 1$

$$\frac{dy}{dx} = \frac{6t^2}{6t} = t$$

Point $(4,3)$
$m = 1$
$\boxed{y - 3 = 1(x - 4)}$

Point $(4,3)$
$m = -1$
$\boxed{y - 3 = -1(x - 4)}$