

Calculus

Area of Polar Functions

$r = 4 + 3\cos\theta$	θ
7	0
4	$\pi/2$
1	π
4	$3\pi/2$
7	2π

Name _____

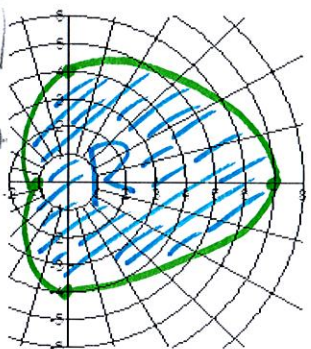
Date _____ Pd. _____

For each sketch the area of the region and then find the area.

1. Find the area of the region bounded by $r = 4 + 3\cos\theta$ on the interval $(0, 2\pi)$.

$$\frac{1}{2} \int_0^{2\pi} (4 + 3\cos\theta)^2 d\theta = \frac{1}{2} (128.805)$$

$$\text{OR} = \boxed{64.403}$$

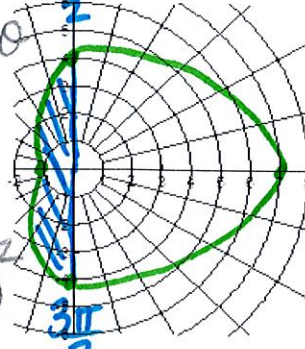
$$2 \left(\frac{1}{2} \int_0^{\pi} (4 + 3\cos\theta)^2 d\theta \right) = \boxed{64.403}$$


2. Find the area of the region bounded by on the interval $(\frac{\pi}{2}, \frac{3\pi}{2})$.

$$\frac{1}{2} \int_{\pi/2}^{3\pi/2} (4 + 3\cos\theta)^2 d\theta$$

$$\frac{1}{2} [16.403]$$

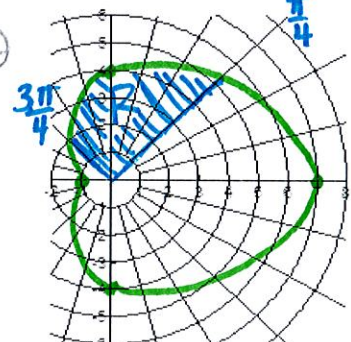
$$\text{OR} = \boxed{8.201}$$

$$2 \left(\frac{1}{2} \int_{\pi/2}^{\pi} (4 + 3\cos\theta)^2 d\theta \right) = \boxed{8.201}$$


3. Find the area of the region bounded by $r = 4 + 3\cos\theta$ on the interval $(\frac{\pi}{4}, \frac{3\pi}{4})$.

$$\frac{1}{2} \int_{\pi/4}^{3\pi/4} (4 + 3\cos\theta)^2 d\theta$$

$$\frac{1}{2} [27.701]$$

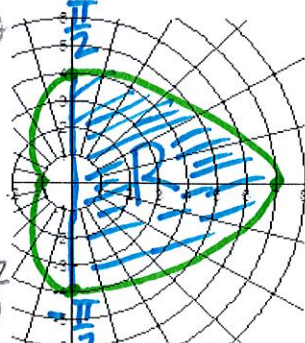
$$\boxed{13.851}$$


4. Find the area of the region bounded by $r = 4 + 3\cos\theta$ on the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$.

$$\frac{1}{2} \int_{-\pi/2}^{\pi/2} (4 + 3\cos\theta)^2 d\theta$$

$$\frac{1}{2} [112.403]$$

$$\boxed{56.201}$$

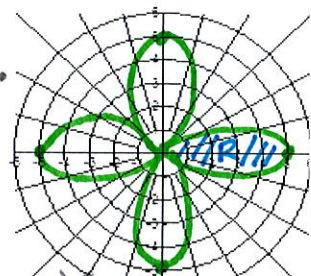
$$\text{OR} = 2 \left(\frac{1}{2} \int_0^{\pi/2} (4 + 3\cos\theta)^2 d\theta \right) = \boxed{56.201}$$


5. Find the area of one petal of $r = 5\cos(2\theta)$.

r	θ
5	0
0	$\pi/4$
-5	$\pi/2$
0	$3\pi/4$
5	π
0	$5\pi/4$
-5	$3\pi/2$
0	$7\pi/4$
5	2π

$$\int_{-\pi/4}^{\pi/4} \text{OR} \int_{3\pi/4}^{5\pi/4} \text{OR} \dots$$

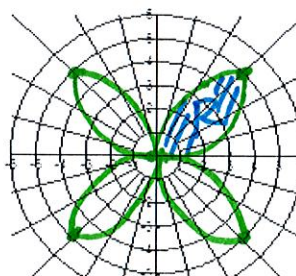
$$\frac{1}{2} \int_{-\pi/4}^{\pi/4} (5\cos(2\theta))^2 d\theta$$

$$\frac{1}{2} [19.635] = \boxed{9.817}$$


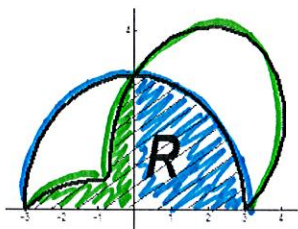
6. Find the area of one petal of $r = 5\sin(2\theta)$.

r	θ
0	0
5	$\pi/4$
0	$\pi/2$
-5	$3\pi/4$
0	π
5	$5\pi/4$
0	$3\pi/2$
-5	$7\pi/4$
0	2π

$$\frac{1}{2} \int_0^{\pi/2} (5\sin(2\theta))^2 d\theta$$

$$\frac{1}{2} [19.635] = \boxed{9.817}$$


1. The graphs of the polar curves $r=3$ and $r=3+2\sin(2\theta)$ are shown in the figure to the right for $0 \leq \theta \leq \pi$. Let



R be the shaded region that is inside the graph of $r=3$ and inside the graph of $r=3+2\sin(2\theta)$. Find the area of R.

$$3 = 3 + 2\sin(2\theta)$$

$$0 = 2\sin(2\theta) \quad \text{Intersect}$$

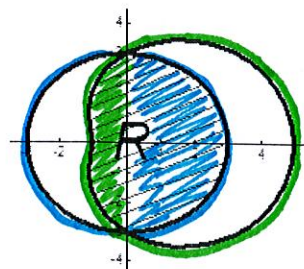
$$\sin(2\theta) = 0 \quad \theta = \frac{\pi}{2}$$

$$\frac{1}{2} \int_{\pi/2}^{\pi} (3+2\sin(2\theta))^2 + \frac{1}{4}\pi(3)^2$$

$$\frac{1}{2} [5.279] + \frac{9\pi}{4}$$

$$2.639 + 7.068 = 9.707$$

2. The graphs of the polar curves $r=3$ and $r=3+2\cos(\theta)$ are shown in the figure to the right. Let R be the region that is



inside the graph of $r=3$ and also inside the graph of $r=3+2\cos(\theta)$, as shaded in the figure. Find the area of R.

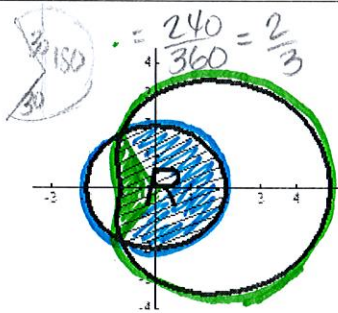
$$\frac{1}{2} \int_{\frac{3\pi}{2}}^{\frac{\pi}{2}} (3+2\cos\theta)^2 + \frac{1}{2}\pi(3)^2$$

$$\frac{1}{2} [10.558] + \frac{9\pi}{2}$$

$$5.279 + 14.137$$

$$19.416$$

3. The graphs of polar curves $r=2$ and $r=3+2\cos(\theta)$ are shown in the figure to the right. Let R be the region



that is inside the graph of $r=2$ and also inside the graph of $r=3+2\cos(\theta)$, as shaded in the figure. Find the area of R.

$$2 = 3 + 2\cos\theta$$

$$-1 = 2\cos\theta$$

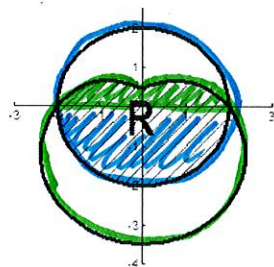
$$\cos\theta = -\frac{1}{2} \quad \theta = \frac{2\pi}{3} \text{ \& } \frac{4\pi}{3}$$

$$\frac{1}{2} \int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} (3+2\cos\theta)^2 d\theta + \frac{2}{3}\pi(2)^2$$

$$\frac{1}{2} [3.986] + \frac{8\pi}{3}$$

$$10.370$$

4. The graphs of polar curves $r=2$ and $r=2-\frac{3}{2}\sin(\theta)$ are shown in the figure to the right. Let R be the region



that is inside the graph of $r=2$ and also inside the graph of $r=2-\frac{3}{2}\sin(\theta)$, as shaded in the figure. Find the area of R.

$$2 = 2 - \frac{3}{2}\sin\theta$$

$$0 = -\frac{3}{2}\sin\theta$$

$$\sin\theta = 0 \quad \theta = 0, \pi$$

$$\frac{1}{2} \int_0^{\pi} (2-\frac{3}{2}\sin\theta)^2 d\theta + \frac{1}{2}\pi(2)^2$$

$$\frac{1}{2} [4.101] + 2\pi$$

$$8.336$$