AP Calculus BC

Name________________________

Area of Polar Functions

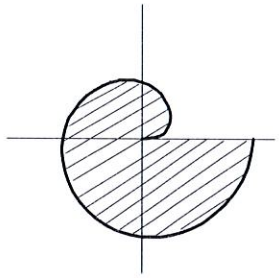
P, P, & V Day 10

1-4: Find the area of the shaded region.

1. $r=\sqrt{\theta}$

$$\frac{1}{2}\int_0^{2\pi}\left[\sqrt{\theta}\right]^2 d\theta$$

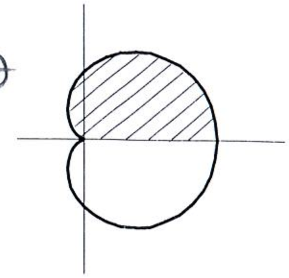
$$\boxed{\pi^2}$$



2. $r=1+\cos\theta$

$$\frac{1}{2}\int_0^{\pi}\left[1+\cos\theta\right]^2 d\theta$$

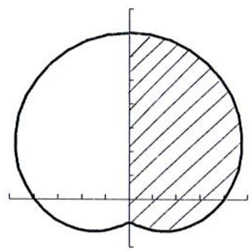
$$\boxed{\frac{3\pi}{4}}$$



3. $r=4+3\sin\theta$

$$\frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left[4+3\sin\theta\right]^2 d\theta$$

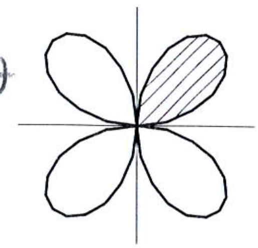
$$\boxed{\frac{41\pi}{4}}$$



4. $r=\sin(2\theta)$

$$\frac{1}{2}\int_0^{\frac{\pi}{2}}\left[\sin(2\theta)\right]^2 d\theta$$

$$\boxed{\frac{\pi}{8}}$$



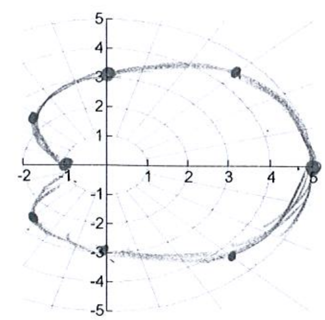
Sketch the curve and find the area that it encloses.

5. $r=3+2\cos\theta$

$r=3+2\cos\theta$	θ
5	0
$3+\sqrt{2}\approx 4.4$	$\frac{\pi}{4}$
3	$\frac{\pi}{2}$
$3-\sqrt{2}\approx 1.6$	$\frac{3\pi}{4}$
1	π

$$2\left[\frac{1}{2}\int_0^{\pi}\left[3+2\cos\theta\right]^2 d\theta\right]$$

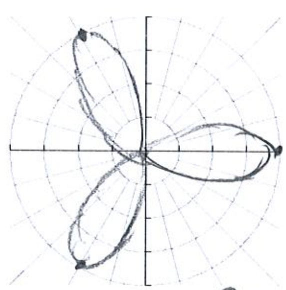
$$\boxed{11\pi}$$



Find the area of the region enclosed by one loop of the curve.

6. $r=4\cos 3\theta$

$r=4\cos 3\theta$	θ
4	0
0	$\pi/6$
-4	$\pi/3$
0	$\pi/2$
4	$2\pi/3$
0	$5\pi/6$
-4	π



$$\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\left[4\cos 3\theta\right]^2 d\theta$$

$$\boxed{\frac{4\pi}{3}}$$

7. $r=1+2\sin\theta$ (inner loop)

$r=0$?

$0=1+2\sin\theta$

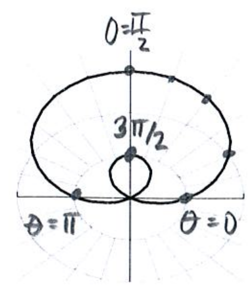
$2\sin\theta=-1$

$\sin\theta=-1/2$

$\theta=\frac{7\pi}{6}$ & $\frac{11\pi}{6}$

$$\frac{1}{2}\int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}}\left[1+2\sin\theta\right]^2 d\theta=$$

$$\boxed{\pi-\frac{3\sqrt{3}}{2}}$$



AP Calculus BC — Name________________________

Area of Polar Functions — P, P, & V Day 10

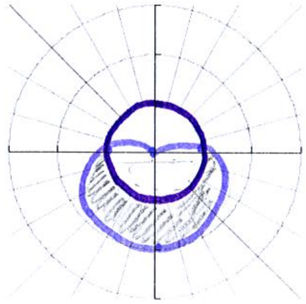
Find the area of the region that lies inside the first curve and outside the second curve.

8. $r = 1 - \sin\theta$

$r = 1$

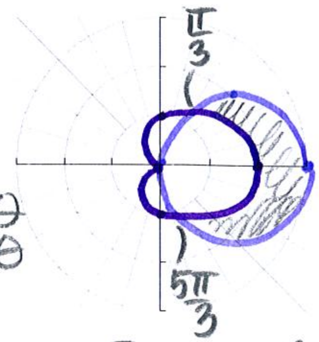
$$\frac{1}{2}\int_{\pi}^{2\pi}[1-\sin\theta]^2\,d\theta - \frac{1}{2}\pi(1)^2$$

$$\boxed{\frac{1}{4}\pi + 2}$$



9. $r = 3\cos\theta$

$r = 1 + \cos\theta$



$3\cos\theta = 1 + \cancel{\cos\theta}$
$-\cos\theta \quad -\cancel{\cos\theta}$

$2\cos\theta = 1$

$\cos\theta = \frac{1}{2}$

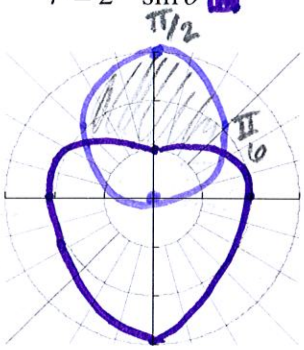
$\theta = \frac{\pi}{3}$ & $\frac{5\pi}{3}$

$$2.\ \frac{1}{2}\int_{0}^{\frac{\pi}{3}}[3\cos\theta]^2 - [1+\cos\theta]^2$$

$$\boxed{\pi}$$

10. $r = 3\sin\theta$

$r = 2 - \sin\theta$



$3\sin\theta = 2 - \sin\theta$
$+\sin\theta \quad +\sin\theta$

$4\sin\theta = 2$

$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}$ & $\frac{5\pi}{6}$

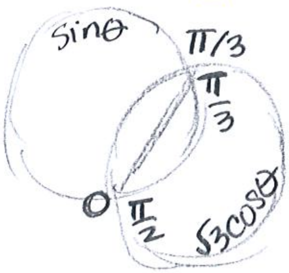
$$2\left[\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}[3\sin\theta]^2 - [2-\sin\theta]^2\right]$$

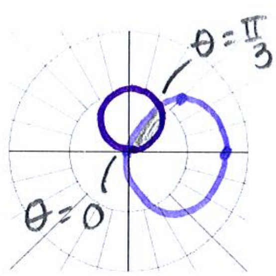
$$\boxed{3\sqrt{3}}$$

Find the area of the region that lies inside both curves.

11. $r = \sqrt{3}\cos\theta$

$r = \sin\theta$





$$\frac{\sqrt{3}\cos\theta}{\cos\theta} = \frac{\sin\theta}{\cos\theta}$$

$\tan\theta = \sqrt{3}$

$\theta = \frac{\pi}{3}$

$$\frac{1}{2}\int_{0}^{\frac{\pi}{3}}[\sin\theta]^2 + \frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}(\sqrt{3}\cos\theta)^2\,d\theta$$

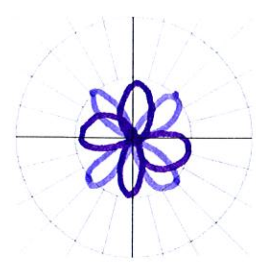
$$\boxed{\frac{5\pi}{24} - \frac{\sqrt{3}}{4}}$$

12. $r = \sin(2\theta)$

$r = \cos(2\theta)$

$\sin 2\theta = \cos 2\theta$

$\theta = \frac{\pi}{8}$



$$8\left[2\left[\frac{1}{2}\int_{0}^{\frac{\pi}{8}}[\sin(2\theta)]^2\right]\right]$$

$$\boxed{\frac{\pi}{2} - 1}$$