AP Calculus-AB

Limits, Continuity, & R.O.C Day 9

Notes: Approximating Instantaneous Rate of Change

R.O.C.

Terminology and Notation:

The *derivative* of f at $x=a$ is the instantaneous rate of change of f at $x=a$. The following notations are used for the derivative of f at $x=a$: derivative is a slope

$f'(a)$ $\left.\dfrac{df}{dx}\right|_{x=a}$ $\text{slope} = \dfrac{\text{Rise}}{\text{Run}} = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

Example(s) 1: Use the given table to approximate the expressions below.

x	-2	-1	0	1	2
$f(x)$	-10	-7	0	2	6
$g(x)$	3	1	-4	-2	5

A.) $f'(-1) \approx \boxed{5}$

$\dfrac{f(0) - f(-2)}{0 - -2}$

$\dfrac{0 - -10}{2} = 5$

B.) $g'(2) \approx \boxed{7}$

$\dfrac{g(2) - g(1)}{2-1}$

$\dfrac{5 - -2}{1} = 7$

C.) $-6g'(1) \approx \boxed{-27}$

$-6\left[\dfrac{g(2) - g(0)}{2-0}\right]$

$-6\left[\dfrac{5 - -4}{2}\right] = -\dfrac{\cancel{6}^{3}}{1}\left[\dfrac{9}{\cancel{2}}\right]$

$= -27$

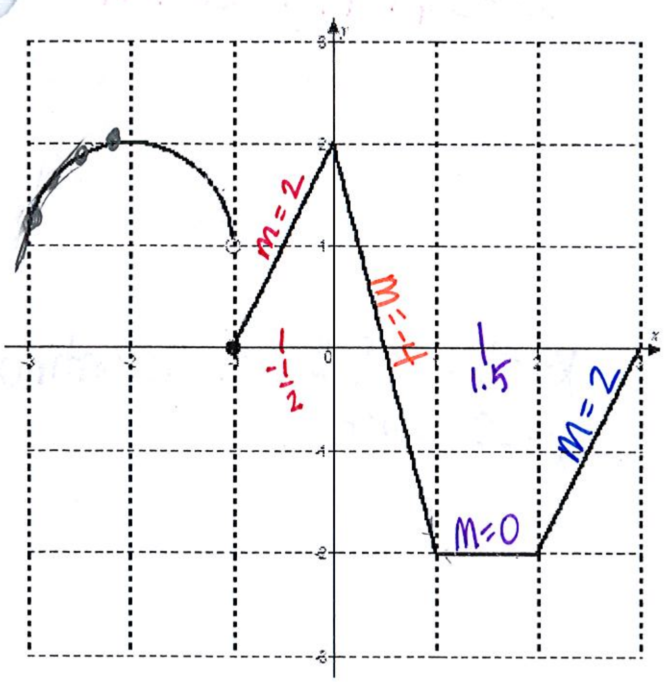
D.) $3g'(0) - f'(2) \approx \boxed{-\dfrac{17}{2} \text{ or } -8.5}$

$3\left[\dfrac{g(1) - g(-1)}{1 - -1}\right] - \left[\dfrac{f(2) - f(1)}{2-1}\right]$

$3\left[\dfrac{-2-1}{2}\right] - \left[\dfrac{6-2}{1}\right]$

$-\dfrac{9}{2} - \dfrac{4 \cdot (2)}{2} = -\dfrac{17}{2}$

Example(s) 2: Use the graph of f below to approximate each derivative.



A.) $f'(-2) = 0$

Slope at max/min = 0

B.) $f'(-1) =$ does not exist

C.) $f'\left(-\dfrac{1}{2}\right) = \boxed{2}$

D.) $f'(0) =$ does not exist

E.) $f'\left(\dfrac{1}{2}\right) = \boxed{-4}$

F.) $f'(1) =$ does not exist

G.) $f'\left(\dfrac{3}{2}\right) = \boxed{0}$

H.) $f'(2) =$ does not exist

I.) $f'\left(\dfrac{5}{2}\right) = \boxed{2}$

$f'(a) = \lim\limits_{\Delta x \to 0} \text{slope}$

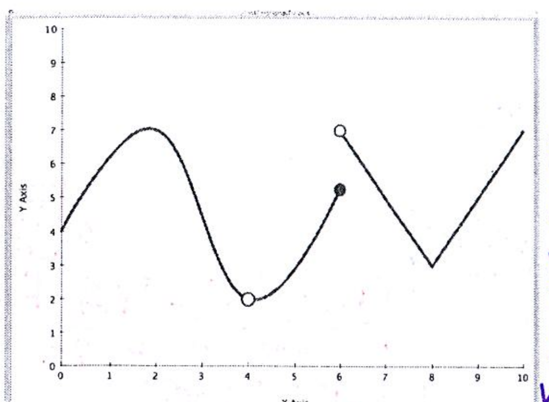
When a function does not have f'(a).

Differentiability:

We say that a function is differentiable at a point if a derivative is defined at the point. A function f will fail to be differentiable at $x=a$ if

- f is discontinuous at $x=a$,
- f has a cusp or corner at $x=a$, or
- f has a vertical tangent at $x=a$

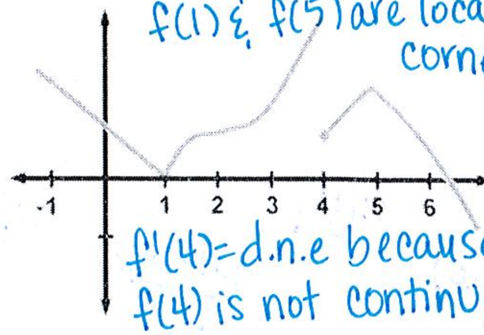
Example(s) 3: State where each function is not differentiable and why.



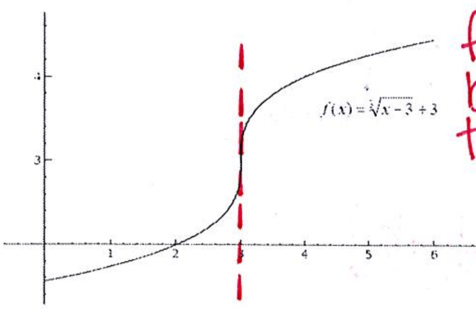
$f'(4)=f'(6)=$ does not exist because $f(4)$ & $f(6)$ are not continuous.

$f'(8)=$ d.n.e. b.c. corner at $x=8$.

$f'(1)=f'(5)=$ d.n.e because $f(1)$ & $f(5)$ are located at corners



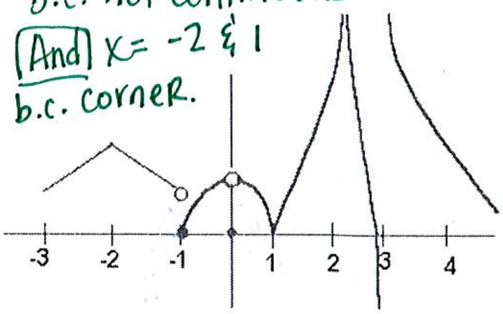
$f'(4)=$ d.n.e because $f(4)$ is not continuous.



$f'(3)=$ d.n.e. because vertical tangent at $x=3$

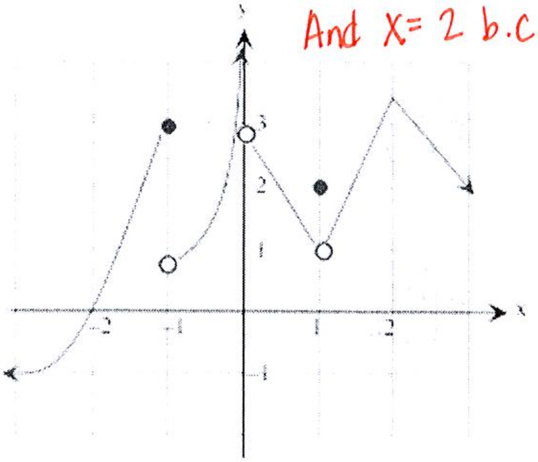
Not differentiable at $x=-1,0,2$ & 3 b.c. not continuous

$\boxed{\text{And}}$ $x=-2$ & 1 b.c. corner.



Not diff. at $x=-1,0,1$ b.c. not cont.

And $x=2$ b.c. corner.



Not diff $x=-3,-1,2$ b.c. not cont.

And $x=-2$ b.c. corner.