AP Calculus-AB

Notes: Special Trigonometric Limits & Intermediate Value Theorem (IVT) Day 8

Two Special Trig. Limits:

$\lim_{x\to 0}\frac{\sin x}{x} = \qquad = \lim_{x\to 0}\frac{x}{\sin x}$

$\lim_{x\to 0}\frac{1-\cos x}{x} = \qquad = \lim_{x\to 0}\frac{\cos x - 1}{x}$

Example(s) 1:

A.) $\lim_{x\to 0}\frac{2\sin x}{x}$

B.) $\lim_{x\to 0}\frac{\sin x}{2x}$

C.) $\lim_{x\to 0}\frac{\sin(2x)}{x}$

D.) $\lim_{x\to 0}\frac{\tan x}{x}$

E.) $\lim_{y\to 0} y\csc y$

F.) $\lim_{\alpha\to\frac{\pi}{2}}\frac{\sin\alpha}{9\alpha}$

Example(s) 2:

A.) $\lim_{x\to 0}\frac{5(1-\cos x)}{x}$

B.) $\lim_{x\to 0}\frac{1-\cos x}{5x}$

C.) $\lim_{x\to 0}\frac{1-\cos(2x)}{7x}$

Intermediate Value Theorem (IVT):

Suppose that f is continuous on the closed interval $[a,b]$ and that M is between $f(a)$ and $f(b)$. Then, there exists some value c on the open interval (a,b) such that $f(c)=M$.

Example(s) 3:

Show that the function $g(x)=e^{-4x}$ takes on the value 1 for some value of x on the interval $(-1,2)$.

Example(s) 4:

Show that the function $y = 3x^3 - 4x - 8$ has a root on the interval $(0, 2)$.

Example(s) 5:

Suppose the function f, as given in the table below, is continuous for all real numbers.

x	0	2	4	6	8	10
$f(x)$	–8	2	5	–1	–10	–2

What is the minimum amount of times that $f(x) = -3.5$?

Example(s) 6:

Suppose the function h, as given in the table below, is continuous for all real numbers.

x	0	2	4	6	8	10
$h(x)$	–8	0	1	1	3	–1

Suppose $f(x) = 4 - 2h(x)$. Show that there must be a value n on $4 < n < 10$ such that $f(n) = 5$.