AP Calculus-AB

Notes: Special Trigonometric Limits & Intermediate Value Theorem (IVT)

<u>Two Special Trig. Limits:</u>

$\lim_{x\to 0}\frac{\sin x}{x} = 1 = \lim_{x\to 0}\frac{x}{\sin x} = \lim_{\smile\to 0}\frac{\sin(\smile)}{\smile}$ $\qquad$ $\lim_{x\to 0}\frac{1-\cos x}{x} = 0 = \lim_{x\to 0}\frac{\cos x - 1}{x}$

Example(s) 1:

A.) $\lim_{x\to 0}\frac{2\sin x}{x}$

$2\lim_{x\to 0}\frac{\sin x}{x} = 2(1) = \boxed{2}$

B.) $\lim_{x\to 0}\frac{\sin x}{2x}$

$\frac{1}{2}\lim_{x\to 0}\frac{\sin x}{x} = \frac{1}{2}(1) = \boxed{\frac{1}{2}}$

C.) $\lim_{2x\to 0}\frac{2\sin(2x)}{2x}$

$2\lim_{2x\to 0}\frac{\sin(2x)}{2x} = 2(1) = \boxed{2}$

D.) $\lim_{x\to 0}\frac{\tan x}{x}$

$\lim_{x\to 0}\frac{\sin x}{\cos x}\cdot\frac{1}{x} = \lim_{x\to 0}\frac{\sin x}{x\cos x} = \lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{1}{\cos x}$

$\lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\frac{1}{\cos x}$

$(1)(\frac{1}{1}) = \boxed{1}$

E.) $\lim_{y\to 0} y\csc y = \lim_{y\to 0} y\left(\frac{1}{\sin y}\right)$

$= \lim_{y\to 0}\frac{y}{\sin y}$

$= \boxed{1}$

F.) $\lim_{\alpha\to\frac{\pi}{2}}\frac{\sin\alpha}{9\alpha} = \frac{\sin(\frac{\pi}{2})}{9(\frac{\pi}{2})} = \frac{1}{\frac{9\pi}{2}} = \frac{1\cdot 2}{9\pi}$

$= \boxed{\frac{2}{9\pi}}$

Example(s) 2:

A.) $\lim_{x\to 0}\frac{5(1-\cos x)}{x}$

$5\lim_{x\to 0}\frac{1-\cos x}{x}$

$5(0)$

$\boxed{0}$

B.) $\lim_{x\to 0}\frac{1-\cos x}{5x}$

$\frac{1}{5}\lim_{x\to 0}\frac{1-\cos x}{x}$

$\frac{1}{5}(0)$

$\boxed{0}$

C.) $\lim_{x\to 0}\frac{1-\cos(2x)}{7x}$

$\frac{1}{7}\lim_{2x\to 0}\frac{2(1-\cos(2x))}{2x}$

$\frac{2}{7}\lim_{2x\to 0}\frac{1-\cos(2x)}{2x}$

$\frac{2}{7}(0) = \boxed{0}$

<u>Intermediate Value Theorem (IVT):</u>

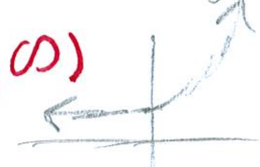
Suppose that f is continuous on the closed interval $[a,b]$ and that M is between $f(a)$ and $f(b)$. Then, there exists some value c on the open interval (a,b) such that $f(c) = M$.

If a function is continuous on $[a,b]$ then your function passes through all the points (y-values) inbetween where you start & finish.

Example(s) 3:

Show that the function $g(x) = e^{-4x}$ takes on the value 1 for some value of x on the interval $(-1,2)$.

$y = e^x$



$g(x) = e^{-4x}$ is exponential so continuous $(-\infty,\infty)$

<u>And</u> that means continuous on $[-1,2]$

$g(-1) = e^4 \approx 54.6$ <u>And</u> $g(2) = e^{-8} \approx .0003$ By IVT $g(x) = 1$ on $(-1,2)$

Example(s) 4:

Showing $y=0$

Show that the function $y=3x^3-4x-8$ has a root on the interval $(0,2)$.

$y=3x^3-4x-8$ is polynomial & continuous on interval $(-\infty,\infty)$
so continuous $[0,2]$
$y(0)=-8$ And $y(2)=8$ By IVT $y(x)=0$ on $(0,2)$

Example(s) 5:

Suppose the function f, as given in the table below, is continuous for all real numbers.

x	0	2	4	6	8	10
$f(x)$	-8	2	5	-1	-10	-2

What is the minimum amount of times that $f(x)=-3.5$?

3 times

Example(s) 6:

Suppose the function h, as given in the table below, is continuous for all real numbers.

x	0	2	4	6	8	10
$h(x)$	-8	0	1	1	3	-1

Suppose $f(x)=4-2h(x)$. Show that there must be a value n on $4<n<10$ such that $f(n)=5$.

$f(x)$ is continuous for all Real numbers.

x	0	2	4	6	8	10
f(x)			2	2	-2	6

Know
$f(x)=4-2h(x)$
$f(4)=4-2h(4)=4-2(1)=2$
$f(6)=4-2h(6)=4-2(1)=2$
$f(8)=4-2h(8)=4-2(3)=-2$
$f(10)=4-2h(10)=4-2(-1)=6$

$f(n)=5$ on $(8,10)$