Day ~~X~~ 8 Application of Integration

Notes: Differential Equations

1-5: Find the ~~par~~ general solution of the differential equation

Ex1 $\frac{dy}{dx} = \frac{x}{y}$

$y\,dy = x\,dx$ $\boxed{1.}$ Seperate Variables

$\int y\,dy = \int x\,dx$ $\boxed{2.}$ Integrate both sides (don't forget +C on right side)

$2\left(\frac{y^2}{2} = \frac{x^2}{2} + C\right)$

$\boxed{3}$ Solve for y=

$\sqrt{y^2} = \sqrt{x^2 + C}$

$y = \pm\sqrt{x^2 + C}$

Ex2 $\frac{dy}{dx} = \frac{x^2+2}{3y^2}$

$\int 3y^2\,dy = \int (x^2+2)\,dx$

$\sqrt[3]{y^3} = \sqrt[3]{\frac{x^3}{3} + 2x + C}$

$y = \sqrt[3]{\frac{1}{3}x^3 + 2x + C}$

Ex3 $yy' - 2e^x = 0$

$y\frac{dy}{dx} - 2e^x = 0$

$\cancel{dx}\left(y\frac{dy}{\cancel{dx}}\right) = 2e^x\,dx$

$\int y\,dy = \int 2e^x\,dx$

$\cancel{2}\left(\frac{y^2}{\cancel{2}}\right) = (2e^x + C)2$

$\sqrt{y^2} = \sqrt{4e^x + C}$

$y = \pm\sqrt{4e^x + C}$

Ex4 $(2+x)y' = 3y$

$$\cancel{dx}\left[(2+x)\frac{dy}{\cancel{dx}}\right] = (3y)\,dx$$

$$\frac{1}{\cancel{2+x}}\cancel{(2+x)}\,dy = 3y\,dx\,\frac{1}{2+x}$$

$$\frac{1}{y}\,dy = \frac{3\cancel{y}}{2+x}\,dx\,\frac{1}{\cancel{y}}$$

$$\int \frac{1}{y}\,dy = \int \frac{3}{2+x}\,dx$$

$$\ln|y| = 3\int \frac{1}{2+x}\,dx \qquad u = 2+x, \; du = dx$$

$$\ln|y| = 3\int \frac{1}{u}\,du$$

$$\cancel{\ln}|y| = 3\ln|2+x| + C$$

$$\cancel{e} \qquad e$$

$$|y| = e^{3\ln|2+x|+C}$$

$$|y| = e^{\ln|(2+x)^3|+C}$$

$$|y| = \cancel{e^{\ln}}|(2+x)^3| \cdot e^C$$

$$|y| = |(2+x)^3| \cdot e^C$$

$$|y| = C|(2+x)^3|$$

$$y = \pm C(2+x)^3$$

$$\boxed{y = C(2+x)^3}$$

Rules of ln

1. $\ln(ab) = \ln a + \ln b$
2. $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$
3. $a\ln b = \ln b^a$
4. $\ln(1) = 0$

Rules of Exp

1. $x^a \cdot x^b = x^{a+b}$

$$x^{☺+C} = x^{☺} \cdot x^C$$

$$e^C = C$$

$$|x| = 4 \rightarrow x = 4, \; x = -4$$

$$x = \pm 4$$

$$\pm C = C$$

Ex5 $y \cdot \ln x - xy' = 0$

$$y\ln x - x \cdot \frac{dy}{dx} = 0$$

$$y\ln x = x\frac{dy}{dx}$$

$$\cancel{dx}\left(x\frac{dy}{\cancel{dx}}\right) = (y\ln x)\,dx$$

$$\frac{1}{y} \cdot \frac{1}{\cancel{x}}\cancel{x}\,dy = \frac{1}{\cancel{x}}\cancel{y}\ln x\,dx\,\frac{1}{\cancel{y}}$$

$$\int \frac{1}{y}\,dy = \int \frac{\ln x \cdot 1}{\cancel{x}}\,\cancel{dx} \qquad u = \ln x, \; du = \frac{1}{x}\,dx$$

$$\ln|y| = \int u\,du$$

$$\ln|y| = \frac{u^2}{2} + C$$

$$\cancel{\ln}|y| = \frac{1}{2}(\ln x)^2 + C$$

$$\cancel{e} \qquad e$$

$$|y| = e^{\frac{1}{2}(\ln x)^2 + C}$$

$$|y| = e^{\frac{1}{2}(\ln x)^2} e^C$$

$$|y| = Ce^{\frac{1}{2}(\ln x)^2}$$

$$y = \pm Ce^{\frac{1}{2}(\ln x)^2}$$

$$y = Ce^{\frac{1}{2}(\ln x)^2}$$

6-7: Find the particular solution of the differential equation that satisfies the initial condition.

Ex6 $yy' - e^x = 0$ given $y(0)=4$

$$y\frac{dy}{dx} - e^x = 0$$

$$\cancel{dx}\left(y\frac{dy}{\cancel{dx}}\right) = e^x\,dx$$

$$\int y\,dy = \int e^x\,dx$$

$$\frac{y^2}{2} = e^x + C$$

$$y(\overset{x}{0}) = \overset{y}{4}$$

$$\frac{(4)^2}{2} = e^0 + C$$

$$8 = 1 + C$$

$$C = 7$$

$$\cancel{2}\left(\frac{y^2}{\cancel{2}}\right) = (e^x + 7)2$$

$$\sqrt{y^2} = \sqrt{2e^x + 14}$$

$$y = \pm\sqrt{2e^x + 14}$$

$$y(0) = 4$$

$$4 = \pm\sqrt{2e^0 + 14}$$

$$4 = \pm\sqrt{16}$$

$$\boxed{y = \sqrt{2e^x + 14}}$$

Ex7 $y(x+1) + \frac{dy}{dx} = 0$ given $y(-2)=1$

$$\cancel{dx}\,\frac{dy}{\cancel{dx}} = -y(x+1)\,dx$$

$$\frac{1}{y}\,dy = -\cancel{y}(x+1)\,dx\,\frac{1}{\cancel{y}}$$

$$\int \frac{1}{y}\,dy = \int -x-1\,dx$$

$$\ln|y| = -\frac{x^2}{2} - x + C \qquad y(\overset{}{\underset{x}{-2}}) = \underset{y}{1}$$

$$\ln|1| = -\frac{(-2)^2}{2} - (-2) + C$$

$$0 = -\frac{4}{2} + 2 + C$$

$$C = 0$$

$$\ln|y| = -\frac{1}{2}x^2 - x + 0$$

$$e^{\ln|y|} = e^{-\frac{1}{2}x^2 - x}$$

$$|y| = e^{-\frac{1}{2}x^2 - x}$$

$$y = \pm e^{-\frac{1}{2}x^2 - x}$$

$$1 = \pm e^{-\frac{1}{2}(-2)^2 - (-2)}$$

$$1 = \pm e^0$$

$$\boxed{y = e^{-\frac{1}{2}x^2 - x}}$$