Notes: Derivatives of General Exponentials and Logarithmic Functions

Review from Algebra

Derivatives Day 8

Rules for Logs:

1. $\log(ab) = \log(a) + \log(b)$
2. $\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$
3. $\log(a^b) = b \cdot \log(a)$

Rules for Logs:

1. $\log(ab) =$
2. $\log\left(\frac{a}{b}\right) =$
3. $\log(a^b) =$

~~Change of Base Formula:~~

~~$\log_b x =$~~

~~Change of Base Formula~~

~~$\log_b x =$~~ __________________

$x^a \cdot x^b = x^{a+b}$

$\frac{x^a}{x^b} = x^{a-b}$

Derivatives of Exponentials:

$\frac{d}{dx}[b^x] = b^x \cdot \ln b$

$\frac{d}{dx}[b^{\cancel{A(x)}}] = b^{AT} \cdot \ln b \cdot \frac{d}{dx}[AT]$

$\frac{d}{dx}[b^x] =$

$\frac{d}{dx}[b^{AT}] =$

$AT = Anything$

Before today

$\frac{d}{dx}[x^2] = 2x$

Today

$\frac{d}{dx}[2^x]$

Derivatives of Natural Logs:

$\frac{d}{dx}[\ln(x)] = \frac{1}{x}$

$\frac{d}{dx}[\ln(\cancel{A(x)})] = \frac{1}{AT} \cdot \frac{d}{dx}[AT]$

$\frac{d}{dx}[\ln x] =$ D

$\frac{d}{dx}[\ln(AT)] =$

$AT = Anything$

$\frac{d}{dx}[e^x] = e^x \cdot \cancel{\ln e}$

Example One: Find the derivative of each: $\frac{d}{dx}[b^{AT}] = b^{AT} \cdot \ln b \cdot \frac{d}{dx}[AT]$

A. $f(x) = 4^{3x}$

$f'(x) = 4^{3x} \cdot \ln 4 \cdot \frac{d}{dx}[3x]$

$f'(x) = 3 \cdot 4^{3x} \cdot \ln 4$

B. $f(x) = 5^{x^2}$

$f'(x) = 5^{x^2} \cdot \ln 5 \cdot \frac{d}{dx}[x^2]$

$f'(x) = 2x \cdot 5^{x^2} \cdot \ln 5$

C. $f(x) = \boxed{x}\boxed{3^x}$

$f'(x) = x \cdot \frac{d}{dx}[3^x] + 3^x \cdot \frac{d}{dx}[x]$

$f'(x) = x \cdot 3^x \ln 3 + 3^x(1)$

$f'(x) = 3^x(x \ln 3 + 1)$

Example Two: Find $f'(x)$ of each:

A. $f(x) = \boxed{x}\boxed{\ln x}$

$f'(x) = x \cdot \frac{d}{dx}[\ln x] + \ln x \frac{d}{dx}[x]$

$f'(x) = \cancel{x} \cdot \frac{1}{\cancel{x}} + \ln x(1)$

$f'(x) = 1 + \ln(x)$

B. $f(x) = (\ln x)^2$ $(AT)^2$

$f'(x) = 2(\ln x)^1 \cdot \frac{d}{dx}(\ln(x))$

$f'(x) = 2 \cdot \ln(x) \cdot \frac{1}{x}$

$f'(x) = \frac{2\ln(x)}{x}$

C. $f(x) = \ln(x^2)$ $\ln(AT)$

$f'(x) = \frac{1}{x^2} \cdot \frac{d}{dx}[x^2]$

$f'(x) = \frac{1}{\cancel{x^2}} \cdot \cancel{2x} = \frac{2}{x}$

$\boxed{1.}\ ln(ab) = ln(a) + ln(b)$
$\boxed{2.}\ ln(\frac{a}{b}) = ln(a) - ln(b)$
$\boxed{3.}\ ln\,a^b = b \cdot ln(a)$

$y = \cancel{ln}(\cancel{e}^{AT}) = AT$
$y = \cancel{e}^{ln(AT)} = AT$

Example Three: Find $\frac{dy}{dx}$ of each:

A. $y = ln(x^2 + 1)$

$y' = \frac{1}{x^2+1} \cdot \frac{d}{dx}[x^2+1]$

$y' = \frac{2x}{x^2+1}$

B. $y = ln(\sqrt{sinx})$

$y' = \frac{1}{\sqrt{sinx}} \cdot \frac{d}{dx}[(sinx)^{1/2}]$

$= \frac{1}{\sqrt{sinx}} \cdot \frac{1}{2}(sinx)^{-1/2} \cdot \frac{d}{dx}[sinx]$

$= \frac{cosx}{2\sqrt{sinx} \cdot \sqrt{sinx}}$

$= \frac{cosx}{2sinx} = \frac{1}{2}cotx$

$y = ln(sinx)^{1/2}$

$y = \frac{1}{2}ln(sinx)$

$y' = \frac{1}{2} \cdot \frac{1}{sinx} \cdot \frac{d}{dx}[sinx]$

$y' = \frac{cosx}{2sinx}$

$y' = \frac{1}{2}cotx$

C. $y = \cancel{ln}(\cancel{e}^x)$

$y = x$

$y' = 1$

$\frac{d}{dx}[log_b x] = \frac{1}{x \cdot lnb}$

$\frac{d}{dx}[log_b(AT)] = \frac{1}{AT \cdot lnb} \cdot \frac{d}{dx}(AT)$

Example Four: Find the derivative of each

A. $y = log_{10}x$

$y' = \frac{1}{x \cdot ln10}$

B. $y = log_3 x$

$y' = \frac{1}{x \cdot ln3}$

C. $y = log_4(x^2 + x)$

$y' = \frac{1}{(x^2+x) \cdot ln4} \cdot \frac{d}{dx}[x^2+x]$

$y' = \frac{2x+1}{ln4(x^2+x)}$

$\boxed{1}\ ln(ab) = lna + lnb$
$\boxed{2}\ ln(\frac{a}{b}) = lna - lnb$
$\boxed{3}\ ln\,a^b = b \cdot lna$

Example Five: ~~Find the derivative of each~~ Expand each

A. $ln\,f(x) = ln\frac{(x+1)^2(2x^2-3)}{\sqrt{x^2+1}}$

$= ln\left(\frac{(x+1)^2 \cdot (2x^2-3)}{(x^2+1)^{1/2}}\right)$

$= ln(x+1)^2 + ln(2x^2-3) - ln(x^2+1)^{1/2}$

$= 2ln(x+1) + ln(2x^2-3) - \frac{1}{2}ln(x^2+1)$

B. $f(x) = \frac{x(x+1)^3}{(3x-1)^2}$

$ln(x) + ln(x+1)^3 - ln(3x-1)^2$

$lnx + 3ln(x+1) - 2ln(3x-1)$

$\frac{d}{dx}[x^2] = 2x$ $\frac{d}{dx}[2^x] = 2^x \cdot \ln 2$

Example 6: Find $f'(x)$ of each:

A. $f(x) = x^x$

B. $f(x) = x^{sinx}$

moved to next test

:)

Example 7: Find the equation of the tangent line to the function at the given point.

A. $f(x) = (\sqrt{2})^x$ $at \quad x = \sqrt{2}$

omit

B. $s(t) = \ln t$ $at \quad \cancel{x}\, t = 5$

$S - \ln 5 = \frac{1}{5}(t-5)$

$S(t) = \ln(t)$

$S'(t) = \frac{1}{t}$

$y - \ln 5 = \frac{1}{5}(x-5)$

Tangent

$\boxed{1}$ Point $s(5) = \ln 5$

$\boxed{2}$ Slope $s'(5) = \frac{1}{5}$

C. $f(x) = \ln(sinx)$ $at \quad x = \frac{\pi}{4}$

$f(x) = \ln(\sin x)$

$f'(x) = \frac{1}{\sin x} \cdot \cos x = \cot x$

$\boxed{1}$ Point $f(\frac{\pi}{4}) = \ln(\sin\frac{\pi}{4}) = \ln(\frac{\sqrt{2}}{2})$

$\boxed{2}$ Slope $f'(\frac{\pi}{4}) = \cot\frac{\pi}{4} = 1$

$y - \ln(\frac{\sqrt{2}}{2}) = 1(x - \frac{\pi}{4})$

$\log_3 1 = x \quad 3^x = 1$

D. $f(x) = log_3x$ $at \quad x = 1$

$\boxed{1}$ Point $f(1) = \log_3 1 = 0$

$\boxed{2}$ Slope $f'(1) = \frac{1}{\ln 3}$

$f(x) = \log_3 x \quad f'(x) = \frac{1}{x \cdot \ln 3}$

$y - 0 = \frac{1}{\ln 3}(x-1)$