Notes: Net or Total Change $\int Rate = amount$ Integration Day 7

$\int velocity \frac{miles}{hour} =$ miles $\int \frac{dollars}{chips} =$ dollars

$\int acceleration \frac{meters}{second^2} =$ $\frac{meters}{second}$ $\int \frac{gallons}{minute} =$ gallons $\int \frac{cars}{hour} =$ cars

Example 1: At 7am, water begins leaking from a tank at a rate of $leaking = 2 + .25t$ gallons per hour (t is the number of hours after $7am$). How much water is lost between $9am$ and $11\ am$?

t=0 7am
t=1 8am
t=2 9am
t=3 10am
t=4 11am

$amount = \int_{t_1}^{t_2} Rate$

$\int_2^4 2 + \frac{1}{4}t\, dt$

$2t + \frac{1}{4}\frac{t^2}{2}\Big|_2^4$

$2t + \frac{1}{8}t^2\Big|_2^4$

$[8 + \frac{1}{8}(16)] - [4 + \frac{1}{8}(4)]$

$8 + 2 - 4 - \frac{1}{2}$

$\boxed{5.5 \text{ gallons}}$

Example 2: The number of cars per hour passing an observation point along a highway is called the rate of traffic flow $q(t)$ in cars per hour.

A. What is $\int_{t_1}^{t_2} q(t)\, dt =$ $\int_{t_1}^{t_2} \frac{cars}{hour} = \#\text{cars from } (t_1, t_2)$

B. The flow rate is recorded at 15-$minute$ intervals between $7:00am$ and $9:00am$. Estimate the number of cars using the highway during this $2-hour$ period by taking the average of the left and right endpoint approximations.

t	7:00	7:15	7:30	7:45	8:00	8:15	8:30	8:45	9:00
$q(t)$	1,044	1,297	1,478	1,844	1,451	1,378	1,155	802	542

$width = \frac{9-7}{8} = \frac{1}{4}$

Area(Rectangle) = width · length

$R_8 = \frac{1}{4}[1297 + 1478 + 1844 + 1451 + 1378 + 1155 + 802 + 542] = 2486.75$

$L_8 = \frac{1}{4}[1044 + 1297 + 1478 + 1844 + 1451 + 1378 + 1155 + 802] =$ 2612.25

$Avg = \frac{R_8 + L_8}{2} = \boxed{2550 \text{ cars}}$

Displacement VS Distance

<u>Displacement vs Total Distance:</u>

Displacement: How far from home (includes pos/neg) from where you started.

$$Displacement = \int_{t_1}^{t_2} v(t)dt = \int_{t_1}^{t_2} \frac{measure\ of\ length}{measure\ of\ time} dt = measure\ of\ length$$

Total Distance: How far you have traveled.

$$Total\ Distance = \int_{t_1}^{t_2} |v(t)|dt = \int_{t_1}^{t_2} \left|\frac{measure\ of\ length}{measure\ of\ time}\right| dt = measure\ of\ length$$

How far are you from Hillgrove = displacement = $\int_0^t v(t)\,dt$

How far you travel = distance = $\int_0^t |v(t)|\,dt$

Example 3: $v(t)$ is the velocity function of your distance from Hillgrove High School.

A. How far are you from the the Grove after 3 $mins$. $\int_0^3 v(t)\,dt = \boxed{4.5\text{ km}}$

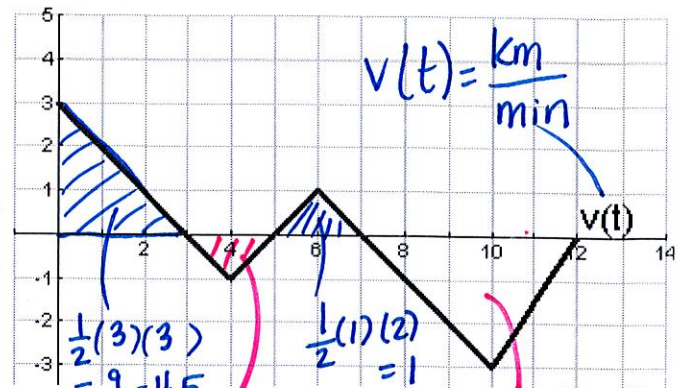
B. How far have you traveled after 3 $mins$. $\int_0^3 |v(t)|\,dt = |4.5| \quad \boxed{4.5\text{ km}}$

C. How far are you from the the Grove after 5 $mins$. $\int_0^5 v(t)\,dt = 4.5 - 1 = \boxed{3.5\text{ km}}$

D. How far have you traveled after 5 $mins$. $\int_0^5 |v(t)|\,dt = |4.5| + |-1| = \boxed{5.5\text{ km}}$

E. How far are you from the the Grove after 1~~3~~2 $mins$. $\int_0^{12} v(t) = 4.5 - 1 + 1 - 7.5 = \boxed{-3\text{ km}}$

F. How far have you traveled after 1~~3~~2 $mins$. $\int_0^{12} |v(t)|\,dt = 4.5 + 1 + 1 + 7.5 = \boxed{14\text{ km}}$



$\frac{1}{2}(3)(3) = \frac{9}{2} = 4.5$

$\frac{1}{2}(1)(2) = 1$

$-\frac{1}{2}(2)(1)$
-1

$-\frac{1}{2}(5)(3)$
-7.5

Example 4: Assume a particle moves along a straight line with given velocity. Find the total displacement and total distance over the time interval.

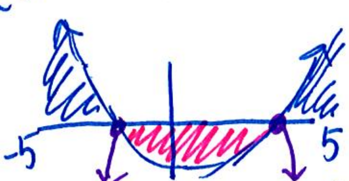
A. $f(x) = x^2 - 2x - 4 \ [-5,5]$

displacement $= \int_{-5}^{5} x^2 - 2x - 4\,dx$

$\frac{x^3}{3} - x^2 - 4x \Big|_{-5}^{5} = \left[\frac{125}{3} - 25 - 20\right] - \left[\frac{-125}{3} - 25 + 20\right]$

$\frac{125}{3} - 45 + \frac{125}{3} + 5 = \frac{250}{3} - \frac{40(3)}{3} = \boxed{\frac{130}{3}}$

distance $= \int_{-5}^{5} |x^2 - 2x - 4|\,dx$



$\boxed{\text{distance} = 73.1476}$

distance $= \int_{-5}^{1.23607} f(x)\,dx - \int_{-1.23607}^{3.23607} f(x)\,dx + \int_{3.23607}^{5} f(x)\,dx = \underline{49.4536} - \underline{14.9071} + \underline{8.78689}$

B. $v(x) = x^2 - x - 6 \ [-10,10]$

displacement $= \int_{-10}^{10} v(x)\,dx = \frac{1640}{3} = \boxed{546.\overline{6}}$

distance $= \int_{-10}^{10} |v(x)|\,dx$

$v(x) = x^2 - x - 6$

$0 = (x-3)(x+2)$

-10 + -2 − 3 + 10

$\boxed{588.\overline{3}}$

$\int_{-10}^{-2} v(x) - \int_{-2}^{3} v(x) + \int_{3}^{10} v(x) = \underline{330.\overline{6}} - \underline{20.8\overline{3}} + \underline{236.8\overline{3}}$